Error-Tolerant Reasoning in the Description Logic $\mathcal{EL}$ Based on Optimal Repairs

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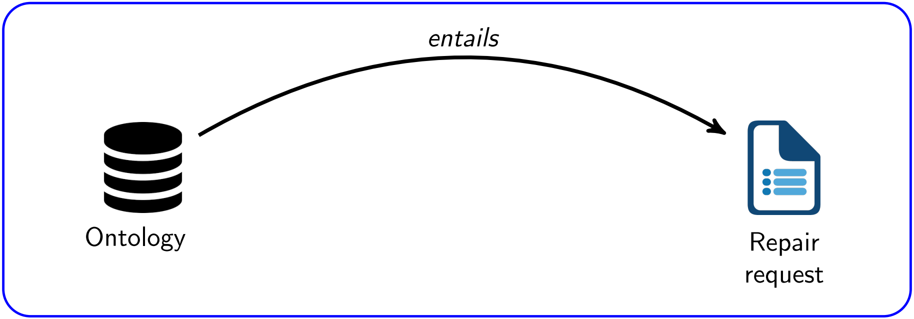
Repairing Description Logic Ontologies

Repairing Ontologies

Ontology = Dataset + Background Knowledge

Repair request = a set of incorrect/unwanted consequences



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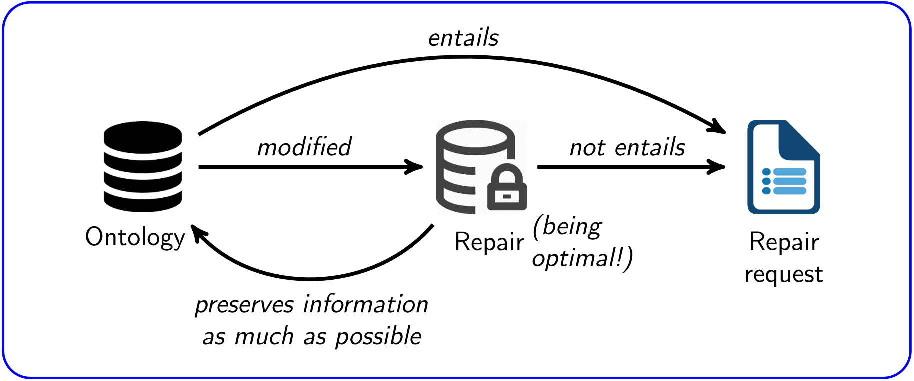

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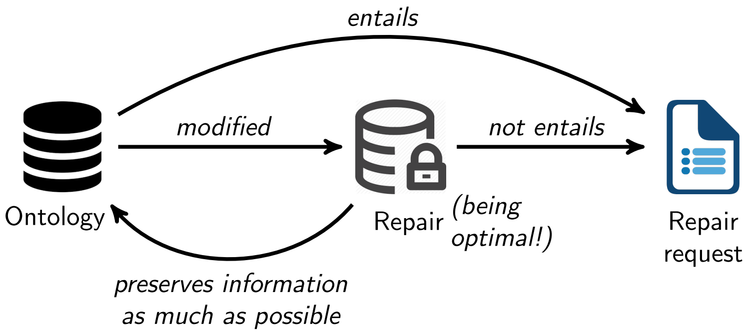
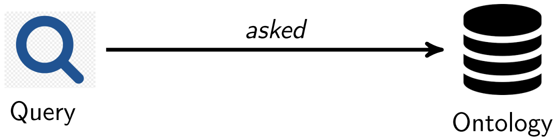


Classical Repairs: preserves a maximal subset of axioms of the ontology

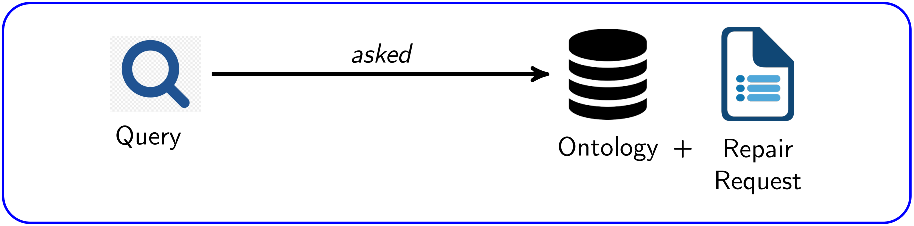
Optimal Repairs: preserves a maximal set of consequences of the ontology

An Illustration on Error-Tolerant Reasoning



Does the ontology return an answer to the query?

Does the ontology entail the query?



Error-Tolerant Reasoning

- Is the query entailed by **some repair** of the ontology? (*brave entailment*)
- Is the query entailed by **each repair** of the ontology? (*cautious entailment*)

Error-Tolerant Reasoning w.r.t. Optimal Repairs

Error-Tolerant Reasoning wr.t. Classical Repairs has been investigated in:

- Ludwig M., Peñaloza R., *Error-Tolerant Reasoning in the Description Logic* $\mathcal{EL}$, JELIA, 2014
- Peñaloza R., *Error-Tolerance and Error Management in Lightweight Description Logics*, KI Journal, 2020

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How about optimal repairs?

Research Questions

- How to perform query reasoning w.r.t. optimal repairs **without computing** the optimal repairs?
- How to **characterize** brave and cautious entailment based on optimal repairs?

Assumption: Our problems are considered in the context of **Description Logics**

How our Dataset Looks Like

Our dataset is a **quantified ABox** $\exists X.\mathcal{A}$
Example: $\exists\{x\}.\{owns(RALF, x), Red(x), Bike(x)\}$

$\exists X.\mathcal{A}$ *is built over*

- X: **variable** names, e.g., $x, y, z, \ldots$
- Σ_C : **concept** names, e.g., *Bike*, *Rider*, ...
- Σ_I: **individual** names, e.g., a,b, *RALF*, *MICK*, ...
- Σ_R : **role** names, e.g., *owns*, *drive*, ...

and the **matrix** $\mathcal{A}$ of the quantified ABox consists of:

- **atomic concept assertions**, e.g., $Rider(RALF), Circuit(x) \ldots$
- **role assertions**, e.g., $drive(RALF, BMW), won(MICK, y) \ldots$

Background Knowledge, Repair Requests, Queries

$\mathcal{EL}$ concepts $C :: \top \mid A \mid \exists r.C \mid C \sqcap C$ $\quad C(a)$ denotes an $\mathcal{EL}$ **concept assertion.**

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$\mathcal{EL}$ concepts $C :: \top \mid A \mid \exists r.C \mid C \sqcap C$ $\quad C(a)$ denotes an $\mathcal{EL}$ **concept assertion.**

The **background knowledge** is represented by an $\mathcal{EL}$ **TBox** e.g.,

$$\mathcal{T} = \{Champion \sqsubseteq Famous,\ \exists drive.RacingCar \sqsubseteq RacingDriver\}$$

Repair requests $\mathcal{R}$ and **Queries** $\mathcal{Q}$ are finite sets of $\mathcal{EL}$ **concept assertions**

$$\mathcal{R} = \{\,(RacingDriver \sqcap Famous)(RALF),\ (\exists father.(RacingDriver \sqcap Famous))(MICK)\}$$

Atoms$(\mathcal{R}, \mathcal{T})$ is a set of $\mathcal{EL}$ **atoms** (**concept names** or **existential restrictions**) occurring in $\mathcal{R} \cup \mathcal{T}$ e.g.,

$$\text{Atoms}(\mathcal{R}, \mathcal{T}) = \{\,Famous, RacingDriver, Champion, RacingCar,\ \exists father.(RacingDriver \sqcap Famous), \exists drive.RacingCar\}$$

Reasoning in $\mathcal{EL}$ with Quantified ABoxes

Reasoning in $\mathcal{EL}$

- $C \sqsubseteq^{\mathcal{T}} D$ means the concept C is **subsumed by** the concept D w.r.t. $\mathcal{T}$
- $\exists X.\mathcal{A} \models^{\mathcal{T}} C(b)$ means that the individual b is an **instance of** the $\mathcal{EL}$ concept C w.r.t. $\exists X.\mathcal{A}$ and $\mathcal{T}$.
- **Subsumption** and **Instance relationships** in $\mathcal{EL}$ can be checked in polynomial time

IQ-Entailment

Interested **only in instance relationships** entailed by the given qABox and TBox

- $\exists X.\mathcal{A}$ IQ-**entails** $\exists Y.\mathcal{B}$ w.r.t. $\mathcal{T}$, denoted by $\exists X.\mathcal{A} \models^{\mathcal{T}}_{\mathsf{IQ}} \exists Y.\mathcal{B}$, if $\exists Y.\mathcal{B} \models^{\mathcal{T}} C(b)$ implies $\exists X.\mathcal{A} \models^{\mathcal{T}} C(b)$ for each concept assertion $C(b)$
- IQ-**entailment** between quantified ABoxes is in P. [CADE '21]

(Optimal) IQ-repairs

IQ-Repairs

Given $\exists X.\mathcal{A}, \mathcal{T}$, and a repair request $\mathcal{R}$,

- the qABox $\exists Y.\mathcal{B}$ is an **IQ-repair** of $\exists X.\mathcal{A}$ for $\mathcal{R}$ w.r.t. $\mathcal{T}$ if
  - $\exists X.\mathcal{A} \models^{\mathcal{T}}_{\mathsf{IQ}} \exists Y.\mathcal{B}$ and
  - for each $C(b) \in \mathcal{R}$, $\exists Y.\mathcal{B} \not\models^{\mathcal{T}} C(b)$.
- $\exists Y.\mathcal{B}$ is **optimal** if there is no IQ-repair $\exists Z.\mathcal{C}$ that strictly IQ-entails $\exists Y.\mathcal{B}$ w.r.t. $\mathcal{T}$

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(Optimal) IQ-repairs may have **exponential size** [CADE '21]

*Are there any **polynomial-size representations** that correspond to (optimal) IQ-repairs such that reasoning with them is **tractable**?*

Seed Functions

Repair Seed Function (rsf)

- It specifies a (superset of) optimal repairs
- It assigns to each individual b a **repair type** $\mathcal{K}$ for b consisting of **atoms that should not hold for b in the repair** such that

  if $P(b) \in \mathcal{R}$ with $\exists X.\mathcal{A} \models^{\mathcal{T}} P(b)$, then there is $D \in \mathcal{K}$ that subsumes P.

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Canonical IQ-Repairs [CADE '21]

- each rsf s induces a **canonical** IQ-**repair**, denoted as $\text{rep}_{\text{IQ}}^{\mathcal{T}}(\exists X.\mathcal{A}, s)$.
- each IQ-repair is IQ-entailed by a canonical IQ-repair
- the set of all canonical IQ-repairs **contains** all optimal IQ-repairs.

Reasoning with Seed Functions

$\mathsf{rep}^{\mathcal{T}}_{\mathsf{IQ}}(\exists X.\mathcal{A}, s) \models^{\mathcal{T}} C(b)$ iff $\exists X.\mathcal{A} \models^{\mathcal{T}} C(b)$ and the repair type $s(b)$ does not contain any atom subsuming C w.r.t. $\mathcal{T}$.

Instance problem w.r.t. canonical IQ-repairs

Given an rsf s and an assertion $C(b)$, we can decide **in polynomial time** whether $\mathsf{rep}^{\mathcal{T}}_{\mathsf{IQ}}(\exists X.\mathcal{A}, s) \models^{\mathcal{T}} C(b)$ **without computing** $\mathsf{rep}^{\mathcal{T}}_{\mathsf{IQ}}(\exists X.\mathcal{A}, s)$.

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What seed functions that induce optimal IQ-repairs?

$\leq$-Minimal Seed Functions

Some notions and results from [CADE '21, ESWC '22] ...

- A repair type $\mathcal{K}$ is **covered by** a repair type $\mathcal{L}$ (denoted as $\mathcal{K} \leq \mathcal{L}$) iff for each $C \in \mathcal{K}$, there is $D \in \mathcal{L}$ such that C is subsumed by D
- s is **covered by** t (denoted as $s \leq t$) if $s(a) \leq t(a)$ for each $a \in \Sigma_\mathsf{I}$

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$$s \leq t \text{ iff } \mathrm{rep}^{\mathcal{T}}_{\mathsf{IQ}}(\exists X.\mathcal{A}, s) \models^{\mathcal{T}}_{\mathsf{IQ}} \mathrm{rep}^{\mathcal{T}}_{\mathsf{IQ}}(\exists X.\mathcal{A}, t)$$

- If ***s* is $\leq$-minimal**, then $\mathrm{rep}^{\mathcal{T}}_{\mathsf{IQ}}(\exists X.\mathcal{A}, s)$ is an optimal IQ-repair
- Every optimal IQ-repair is IQ-**equivalent** to $\mathrm{rep}^{\mathcal{T}}_{\mathsf{IQ}}(\exists X.\mathcal{A}, s)$ for a $\leq$-minimal rsf s.

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There is a **naïve procedure** for deciding the $\leq$-minimality of seed function running in exponential time.

Can we decide the $\leq$-minimality of seed functions in polynomial time?

Deciding $\leq$-Minimality of Seed Functions

$\leq$-Minimality of Seed Functions is in P **(Idea)**

If s is not $\leq$-minimal, then there is an rsf t such that $t < s$.

If $t < s$, then there exist $b \in \Sigma_\mathsf{I}$ and $D \in s(b)$ such that no atom in $t(b)$ that subsumes D.

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From $s(b)$, we compute a repair type $\mathcal{L}_b$ for b such that

- $t(b) \leq \mathcal{L}_b < s(b)$ and $\mathcal{L}_b$ **does not contain** D,
- if $P(b) \in \mathcal{R}$ with $\exists X.\mathcal{A} \models^\mathcal{T} P(b)$, then there exists an atom in $\mathcal{L}_b$ that subsumes P.

Computing such a repair type $\mathcal{L}_b$ for each $b \in \Sigma_\mathsf{I}$ can be done in **polynomial time**.

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What can we do with $\leq$-minimal seed functions for dealing with error-tolerant reasoning problems?

Error-Tolerant Reasoning w.r.t. Optimal IQ-Repairs

Brave Entailment

A query $\mathcal{Q}$ is bravely entailed by $\exists X.\mathcal{A}$ for $\mathcal{R}$ w.r.t.$\mathcal{T}$ iff **there is** an optimal IQ-repair $\exists Y.\mathcal{B}$ such that $\exists Y.\mathcal{B} \models^{\mathcal{T}} C(a)$ for each $C(a) \in \mathcal{Q}$.

Cautious Entailment

A query $\mathcal{Q}$ is cautiously entailed by $\exists X.\mathcal{A}$ for $\mathcal{R}$ w.r.t. $\mathcal{T}$ iff **every** optimal IQ-repair $\exists Y.\mathcal{B}$ satisfies $\exists Y.\mathcal{B} \models^{\mathcal{T}} C(a)$ for each $C(a) \in \mathcal{Q}$.

Note:

- If there is no repair, then **every consequence** is cautiously entailed
- Thus, we require only repair requests that have a repair, namely ...
- Repair requests that are solvable w.r.t. $\mathcal{T}$, i.e., for each $C(a) \in \mathcal{R}$, C is **not tautology w.r.t.** $\mathcal{T}$

Brave Entailment

Brave Entailment is in P

Brave entailment can be **reduced to the instance problem** in $\mathcal{EL}$.

$\mathcal{Q}$ is bravely entailed by $\exists X.\mathcal{A}$ for $\mathcal{R}$ w.r.t. $\mathcal{T}$ iff

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What can we do more with brave entailment?

An (Optimal) Repair for Wanted Consequences

Brave entailment can be used to check in polynomial time whether there exists an IQ-repair $\exists Y.\mathcal{B}$ such that

$\exists Y.\mathcal{B}$ entails **consequences**/query $\mathcal{Q}$ that one **wants to retain**.

Brave Entailment

Brave Entailment is in P

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$\mathcal{Q}$ is bravely entailed by $\exists X.\mathcal{A}$ for $\mathcal{R}$ w.r.t. $\mathcal{T}$ iff the **translation of** $\mathcal{Q}$ into a qABox $\exists Y.\mathcal{B}$ is an IQ-repair of $\exists X.\mathcal{A}$ for $\mathcal{R}$ w.r.t. $\mathcal{T}$.

Such an IQ-repair $\exists Y.\mathcal{B}$ need not be optimal in general, but then ...

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Computing One $\leq$-Minimal rsf

- From $\exists Y.\mathcal{B}$, we can compute in **polynomial time** a $\leq$-minimal rsf t such that $\mathsf{rep}^{\mathcal{T}}_{\mathsf{IQ}}(\exists X.\mathcal{A}, t) \models^{\mathcal{T}}_{\mathsf{IQ}} \exists Y.\mathcal{B}$.
- Since t is $\leq$-minimal, $\mathsf{rep}^{\mathcal{T}}_{\mathsf{IQ}}(\exists X.\mathcal{A}, t)$ is optimal and entails $\mathcal{Q}$

Cautious Entailment

Cautious Entailment w.r.t. Non-Empty TBoxes is in coNP

Non-Cautious Entailment: guess a function $s : \Sigma_{\mathsf{I}} \rightarrow \mathcal{P}(\mathsf{Atoms}(\mathcal{R}, \mathcal{T}))$ and then check whether

- s is a repair seed function and is $\leq$-minimal, and
- there is $C(a) \in \mathcal{Q}$ such that $\mathsf{rep}^{\mathcal{T}}_{\mathsf{IQ}}(\exists X . \mathcal{A}, s) \not\models^{\mathcal{T}} C(a)$.

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(Non)-Cautious Entailment w.r.t. an Empty TBox is in P

- $C(a) \in \mathcal{Q}$ is not entailed by some optimal repair iff there is a $\leq$-minimal rsf s s.t. C is subsumed by **some atom** $D \in s(a)$
- If $\mathcal{T} = \emptyset$, then a minimal rsf s, where $s(a)$ **should contain** such an atom D, can be computed in polynomial time.

Conclusions and Future Work

Conclusion:

- Reasoning w.r.t. canonical repairs of exponential size can be performed by considering only seed functions of polynomial size
- Characterized the $\leq$-minimality of seed functions.
- Investigated the complexities of brave and cautious entailment based on optimal repairs

Future Work:

- Is CoNP upper bound for cautious entailment really **tight**?
- Adding **role assertions** in both repair requests and queries
- **Inconsistent-tolerant** reasoning?