Treating Role Assertions as First-Class Citizens in Repair and Error-Tolerant Reasoning

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Repair request = a set of incorrect/unwanted information
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Repairs & Error-Tolerant Reasoning
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Repair request = a set of incorrect/unwanted information

Classical Repairs: preserves a maximal subset of axioms of the ontology
Optimal Repairs: preserves a maximal set of consequences of the ontology
Does the ontology return an answer to the query?

Does the ontology entail the query?
Error-Tolerant Reasoning

- Is the query entailed by **some repair** of the ontology? (*brave entailment*)
- Is the query entailed by **each repair** of the ontology? (*cautious entailment*)
What Have Been Done Before?

Error-Tolerant Reasoning wr.t. Classical Repairs has been investigated in:


Error-Tolerant Reasoning wr.t. Optimal Repairs has been investigated in:

- Baader F., Kriegel F., Nuradiansyah A., *Error-Tolerant Reasoning in the Description Logic $\mathcal{EL}$ Based On Optimal Repairs*, RuleML+RR, 2022

Regarding optimal repairs, the above work only considers repair requests and queries that are finite sets of instance relationships.

What if we treat repair requests and queries as instance relationships + role assertions?
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- Ludwig M., Peñaloza R., Error-Tolerant Reasoning in the Description Logic \( \mathcal{EL} \), JELIA, 2014
- Peñaloza R., Error-Tolerance and Error Management in Lightweight Description Logics, KI Journal, 2020

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What if we treat repair requests and queries as instance relationships + role assertions?
Role Assertions in Repair Requests and Queries

Why are role assertions interesting to be considered?

Role Assertions as Unwanted Consequences

Assume that an ontology consists of

\[
\text{has\_parent}(NICK, MICHAEL), \quad \text{Famous}(MICHAEL), \\
\exists \text{has\_parent}.\text{Famous} \sqsubseteq \text{Rich}
\]

\[
\exists \text{has\_parent}.\text{Famous}(NICK) \text{ and } \text{Rich}(NICK) \text{ are consequences of the ontology.}
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Why are role assertions interesting to be considered?

Role Assertions as Unwanted Consequences

Assume that an ontology consists of

\[ has\_parent(NICK, MICHAEL), \quad Famous(MICHAEL), \quad \exists has\_parent.\text{Famous} \sqsubseteq Rich \]

\[ \exists has\_parent.\text{Famous}(NICK) \text{ and } Rich(NICK) \text{ are consequences of the ontology.} \]

Suppose that Michael is actually not a parent of Mick:

- \[ has\_parent(NICK, MICHAEL) \text{ should be removed from the ontology.} \]

- But removing it also removes \[ \exists has\_parent.\text{Famous}(NICK) \]

and \[ Rich(NICK) \text{ that are not complained by the user.} \]

How to remove unwanted instance relationships and role assertions without removing non-erroneous consequences?
What We Investigate

Research Questions

- How to **compute optimal repairs** w.r.t. repair requests consisting of instance relationships and role assertions?
- How to perform **query reasoning** w.r.t. optimal repairs
- How to **characterize** brave and cautious entailment based on optimal repairs?

Assumption: Our problems are considered in the context of Description Logic $\mathcal{EL}$ Ontologies
\( \mathcal{EL} \) concepts \( C, D \) :: \( \top | A | \exists r.C | C \cap D \),
where \( A \) is an atomic concept name and \( r \) is a role name.

\( \mathcal{EL} \) atoms can be either a concept name \( A \) or an existential restriction \( \exists r.C \).
**EL concepts** $C, D :: \top | A | \exists r.C | C \sqcap D,$
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**Ontology is a pair of $\exists X.A$ and $T$**

- A **quantified ABox** (qABox) $\exists X.A$ consists of
  - A finite set $X$ of **variable names** and
  - A finite set $A$ of **atomic concept assertions** $A(u)$ and **role assertions** $r(u,v)$, where $u$ and $v$ are either variable or individual names.
- A **TBox** $T$ is a finite set of concept inclusions $C \sqsubseteq D$
EL concepts $C, D :: \top | A | \exists r.C | C \sqcap D$, where $A$ is an atomic concept name and $r$ is a role name.

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ABox Repair Requests and Queries

A repair request $P$ and a query $Q$ is an EL ABox that is a finite set of concept assertions $C(a)$ and role assertions $r(a, b)$, where $a, b$ are individuals.
**Building Blocks, Ontologies, and Entailments**

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**Entailments**

- **ABox-Entailment** (denoted by $\exists X.A \models^T Q$).
- **IRQ-Entailment** (denoted by $\exists X.A \models_{\text{IRQ}}^T \exists Y.B$)
  Interested only in $\mathcal{EL}$ ABoxes entailed by the given qABoxes and TBox.

Both entailments can be checked in **polynomial time** in $\mathcal{EL}$
[CADE’21, ESWC’22]
Given a qABox $\exists X. A$, a TBox $\mathcal{T}$, and an ABox repair request $\mathcal{P}$

- the qABox $\exists Y. B$ is an **IRQ-repair** of $\exists X. A$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ if
  - $\exists X. A \models^\mathcal{T} \exists Y. B$ and
  - no assertion in $\mathcal{P}$ is entailed by $\exists Y. B$ w.r.t. $\mathcal{T}$.

$\exists Y. B$ is **optimal** if there is no IRQ-repair that strictly IRQ-entails $\exists Y. B$ w.r.t. $\mathcal{T}$.
Given a qABox $\exists X . A$, a TBox $T$, and an ABox repair request $P$

- the qABox $\exists Y . B$ is an **IRQ-repair** of $\exists X . A$ for $P$ w.r.t. $T$ if
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  - no assertion in $P$ is entailed by $\exists Y . B$ w.r.t. $T$.

- $\exists Y . B$ is **optimal** if there is no IRQ-repair that strictly IRQ-entails $\exists Y . B$ w.r.t. $T$. 
IRQ-Repairs

Given a qABox $\exists X. A$, a TBox $T$, and an ABox repair request $\mathcal{P}$

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ABox repair requests $\mathcal{P}$ are divided into two parts:

- Instance repair request $\mathcal{P}_C$ consisting of all $C(a)$ from $\mathcal{P}$, and
- Role repair request $\mathcal{P}_R$ consisting of all $r(a, b)$ from $\mathcal{P}$. 
Computing (Optimal) IRQ-Repairs for ABox Repair Requests

1. Dealing with Role Repair Requests
   Computing the optimal IRQ-repair $\exists Z . C$ of $\exists X . A$ for $\mathcal{P}_R$ w.r.t. $\mathcal{T}$ in polynomial time.

2. Dealing with Instance Repair Requests
   Computing the set $\mathcal{R}$ of all canonical repairs of $\exists Z . C$ for $\mathcal{P}_C$ w.r.t. $\mathcal{T}$ [ESWC ’22] in exponential time satisfying the following:
   - each IRQ-repair of $\exists X . A$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ is IRQ-entailed by an element in $\mathcal{R}$ w.r.t. $\mathcal{T}$.
   - $\mathcal{R}$ contains all optimal IRQ-repairs of $\exists X . A$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$.
   - There are exponentially many elements in $\mathcal{R}$, each of which is of exponential size.
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1. Dealing with Role Repair Requests
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   Computing the set $\mathcal{R}$ of all canonical repairs of $\exists Z.C$ for $\mathcal{P}_C$ w.r.t. $\mathcal{T}$ [ESWC ’22] in exponential time satisfying the following:
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   - $\mathcal{R}$ contains all optimal IRQ-repairs of $\exists X.A$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$.
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The set of all optimal IRQ-repairs of $\exists X.A$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ can be computed in exponential time.
Brave IRQ-Entailment

**Brave Entailment**

An ABox query $Q$ is **bravely IRQ-entailed** by $\exists X.A$ for $P$ w.r.t. $T$ iff there is an optimal IRQ-repair $\exists Y.B$ of $\exists X.A$ for $P$ w.r.t. $T$ such that $\exists Y.B \models_T Q$.

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**Brave Entailment is in P**

Brave entailment can be **reduced** to the ABox-entailment problem in $\mathcal{EL}$. $Q$ is bravely IRQ-entailed by $\exists X.A$ for $P$ w.r.t. $T$ iff

$$\exists X.A \models_T Q$$ and no assertion in $P$ is entailed by $Q$ w.r.t. $T$
Cautious Entailment

An ABox query $Q$ is **cautiously IRQ-entailed** by $\exists X. A$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ iff every optimal IRQ-repair $\exists Y. B$ of $\exists X. A$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ satisfies $\exists Y. B \models^\mathcal{T} Q$.

Note: Cannot be solved by a reduction to classical reasoning in $\mathcal{EL}$. One could compute all optimal repairs and then perform instance checking w.r.t. each of them, which would be very expensive. The naïve approach solving the instance problem w.r.t. canonical repair runs in exponential time.

Are there any polynomial-size representations of canonical repairs such that reasoning with them is tractable?
### Cautious Entailment

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Cautious Entailment

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\textit{Are there any polynomial-size representations of canonical repairs such that reasoning with them is tractable?}
Repair Seed Function (rsf)

- It assigns to each individual $b$ a set of atoms that should not hold for $b$ in the repair.
- Each seed function $s$ induces a canonical repair denoted by $\text{rep}(\exists X. A, s)$ [CADE’21]
Repair Seed Functions

**Repair Seed Function (rsf)**

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**Reasoning w.r.t. Canonical IQ-repairs**

Given an rsf $s$ and an ABox $Q$, we can decide **in polynomial time** whether $\text{rep}(\exists X. A, s) \models^T Q$ **without computing** $\text{rep}(\exists X. A, s)$.

*What seed functions that induce optimal IRQ-repairs?*
Introducing a pre-order relation $\leq_{\text{IRQ}}$ on seed functions that reflects IRQ-entailment between canonical repairs.
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$s$ is an $\leq_{IRQ}$-**minimal** seed function iff $\text{rep}(\exists Z.C, s)$ is an optimal IRQ-repair.

**Complexity of the $\leq_{IRQ}$-Minimality Problem**

Deciding whether a seed function is $\leq_{IRQ}$-minimal can be done in polynomial time.
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Cautious Entailment is in coNP (unclear if it is coNP-hard)

Non-Cautious Entailment: guess a function $s$ mapping each individual to a set of atoms in the ontology and then check whether

- $s$ is a repair seed function and is $\leq_{IRQ}$-minimal, and
- $\text{rep}(\exists Z. C, s) \not\models_T Q$. 
If we consider classical repairs of qABoxes, then we obtain a matching lower bound for brave and cautious reasoning.

**Brave Entailment is NP-Complete**
- Guessing a subset of $\mathcal{A}$ and check whether it entails $Q$ w.r.t. $\mathcal{T}$ and is maximal.
- Obtaining NP-hardness by a reduction from the problem **monotone 1-in-3-SAT**.

**Cautious Entailment is coNP-Complete**

Considering **Non-Cautious Entailment**
- Guessing a subset of $\mathcal{A}$ and check whether it does not entail $Q$ w.r.t. $\mathcal{T}$ and is maximal.
- Obtaining NP-hardness by a reduction from the problem **path via a node**.
Conclusions and Future Work

Conclusion:
- Computing optimal IRQ-repairs for ABox repair request can be performed in exponential time.
- Investigated the complexities of brave and cautious entailment of qABoxes based on optimal and classical repairs.
- Characterized the $\leq_{\text{IRQ}}$-minimality of seed functions.

Future Work:
- Is CoNP upper bound for cautious IRQ-entailment really tight?
- Considering more variants of repairs for error-tolerant reasoning
- Inconsistent-tolerant reasoning