Treating Role Assertions as First-Class Citizens in Repair and Error-Tolerant Reasoning

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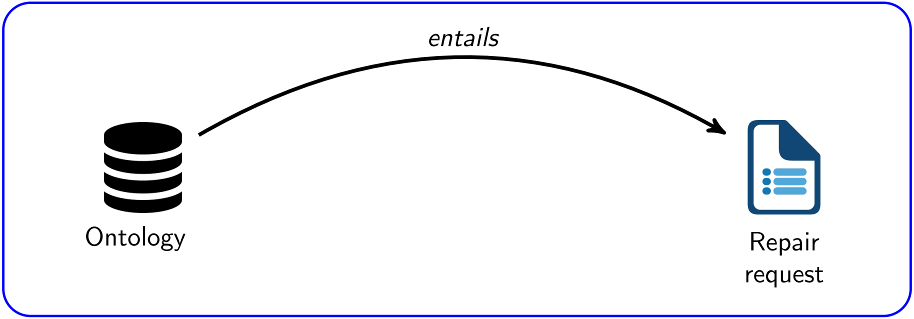
Repair request = a set of incorrect/unwanted information



Repairing Ontologies

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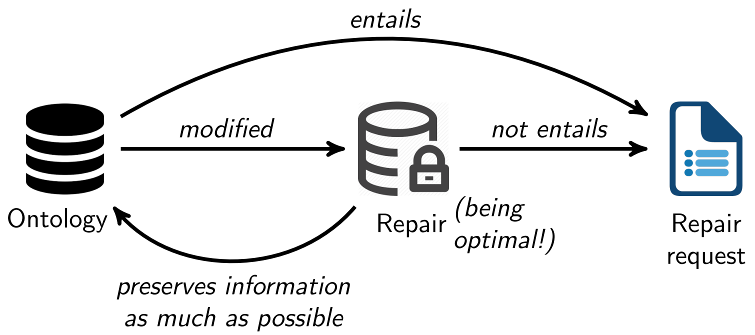

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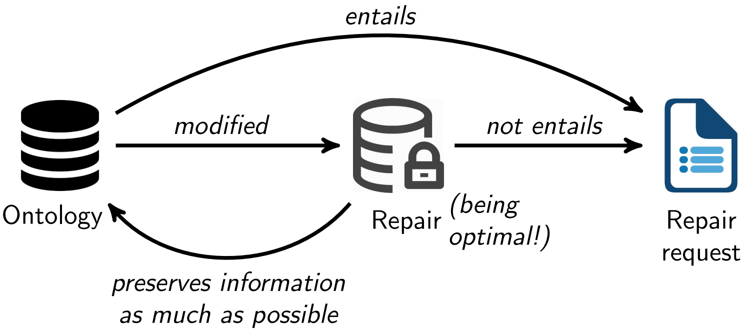
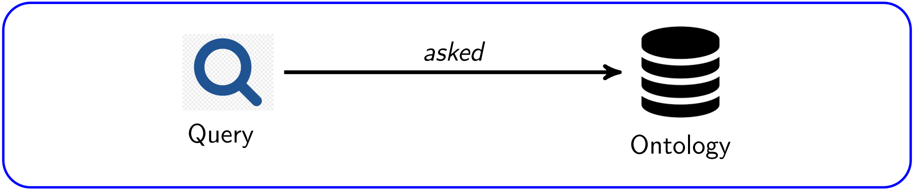


Classical Repairs: preserves a maximal subset of axioms of the ontology

Optimal Repairs: preserves a maximal set of consequences of the ontology

An Illustration on Error-Tolerant Reasoning



Does the ontology return an answer to the query?

Does the ontology entail the query?

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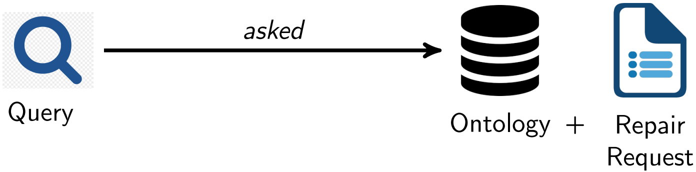

Error-Tolerant Reasoning

- Is the query entailed by **some repair** of the ontology? (*brave entailment*)
- Is the query entailed by **each repair** of the ontology? (*cautious entailment*)

What Have Been Done Before?

Error-Tolerant Reasoning wr.t. Classical Repairs has been investigated in:

- Ludwig M., Peñaloza R., *Error-Tolerant Reasoning in the Description Logic* $\mathcal{EL}$, JELIA, 2014
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Regarding optimal repairs, the above work only considers repair requests and queries that are finite sets of **instance relationships**.

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What if we treat repair requests and queries as

instance relationships + **role assertions**?

Role Assertions in Repair Requests and Queries

Why are role assertions interesting to be considered?

Role Assertions as Unwanted Consequences

Assume that an ontology consists of

$$has\_parent(NICK, MICHAEL), \quad Famous(MICHAEL),$$
$$\exists has\_parent.Famous \sqsubseteq Rich$$

$\exists has\_parent.Famous(NICK)$ and $Rich(NICK)$ are **consequences** of the ontology.

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$\exists has\_parent.Famous(NICK)$ and $Rich(NICK)$ are **consequences** of the ontology.

Suppose that Michael is actually not a parent of Mick:

- $has\_parent(NICK, MICHAEL)$ should be removed from the ontology.
- But removing it **also removes** $\exists has\_parent.Famous(NICK)$ and $Rich(NICK)$ that are not complained by the user.

How to remove unwanted instance relationships and role assertions without removing non-erroneous consequences?

What We Investigate

Research Questions

- How to **compute optimal repairs** w.r.t. repair requests consisting of instance relationships and role assertions?
- How to perform **query reasoning** w.r.t. optimal repairs
- How to **characterize** brave and cautious entailment based on optimal repairs?

Assumption: Our problems are considered in the context of **Description Logic $\mathcal{EL}$ Ontologies**

Building Blocks, Ontologies, and Entailments

$\mathcal{EL}$ **concepts** $C, D :: \top \mid A \mid \exists r.C \mid C \sqcap D$,
where A is an atomic concept name and r is a role name.

$\mathcal{EL}$ **atoms** can be either a concept name A or an existential restriction $\exists r.C$

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Ontology is a pair of $\exists X.\mathcal{A}$ and $\mathcal{T}$

- a **quantified ABox** (qABox) $\exists X.\mathcal{A}$ consists of
  - a finite set X of **variable names** and
  - a finite set $\mathcal{A}$ of **atomic concept assertions** $A(u)$ and **role assertions** $r(u, v)$, where u and v are either variable or individual names.
- a **TBox** $\mathcal{T}$ is a finite set of concept inclusions $C \sqsubseteq D$

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ABox Repair Requests and Queries

A **repair request** $\mathcal{P}$ and a **query** $\mathcal{Q}$ is an $\mathcal{EL}$ ABox that is a finite set of **concept assertions** $C(a)$ and **role assertions** $r(a, b)$, where a, b are individuals.

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Entailments

- **ABox-Entailment** (denoted by $\exists X.\mathcal{A} \models^{\mathcal{T}} \mathcal{Q}$).
- IRQ-**Entailment** (denoted by $\exists X.\mathcal{A} \models^{\mathcal{T}}_{\mathsf{IRQ}} \exists Y.\mathcal{B}$)
  Interested only in $\mathcal{EL}$ ABoxes entailed by the given qABoxes and TBox.

Both entailments can be checked in **polynomial time** in $\mathcal{EL}$
[**CADE'21**, **ESWC'22**]

(Optimal) IRQ-Repairs

IRQ-Repairs

Given a qABox $\exists X.\mathcal{A}$, a TBox $\mathcal{T}$, and an ABox repair request $\mathcal{P}$

- the qABox $\exists Y.\mathcal{B}$ is an **IRQ-repair** of $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ if
  - $\exists X.\mathcal{A} \models^{\mathcal{T}}_{\mathsf{IRQ}} \exists Y.\mathcal{B}$ and
  - no assertion in $\mathcal{P}$ is entailed by $\exists Y.\mathcal{B}$ w.r.t. $\mathcal{T}$.

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- $\exists Y.\mathcal{B}$ is **optimal** if there is no IRQ-repair that strictly IRQ-entails $\exists Y.\mathcal{B}$ w.r.t. $\mathcal{T}$

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ABox repair requests $\mathcal{P}$ are divided into two parts:

- **Instance repair request** $\mathcal{P}_C$ consisting of all $C(a)$ from $\mathcal{P}$, and
- **Role repair request** $\mathcal{P}_R$ consisting of all $r(a, b)$ from $\mathcal{P}$.

Computing (Optimal) IRQ-Repairs for ABox Repair Requests

1. **Dealing with Role Repair Requests**
   Computing the optimal IRQ-repair $\exists Z.\mathcal{C}$ of $\exists X.\mathcal{A}$ for $\mathcal{P}_R$ w.r.t. $\mathcal{T}$ in polynomial time.

2. **Dealing with Instance Repair Requests**
   Computing the set $\mathfrak{R}$ of all **canonical repairs** of $\exists Z.\mathcal{C}$ for $\mathcal{P}_C$ w.r.t. $\mathcal{T}$ [**ESWC '22**] in exponential time satisfying the following:
   - each IRQ-repair of $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ is IRQ-entailed by an element in $\mathfrak{R}$ w.r.t. $\mathcal{T}$.
   - $\mathfrak{R}$ contains all optimal IRQ-repairs of $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$.
   - There are exponentially many elements in $\mathfrak{R}$, each of which is of exponential size.

Computing (Optimal) IRQ-Repairs for ABox Repair Requests

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The set of all optimal IRQ-repairs of $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ can be computed in exponential time.

Brave IRQ-Entailment

Brave Entailment

An ABox query $\mathcal{Q}$ is **bravely IRQ-entailed** by $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t.$\mathcal{T}$ iff **there is** an optimal IRQ-repair $\exists Y.\mathcal{B}$ of $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ such that $\exists Y.\mathcal{B} \models^{\mathcal{T}} \mathcal{Q}$.

Brave Entailment is in P

Brave entailment can be **reduced** to the ABox-entailment problem in $\mathcal{EL}$.

$\mathcal{Q}$ is bravely IRQ-entailed by $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ iff

$$\exists X.\mathcal{A} \models^{\mathcal{T}} \mathcal{Q} \text{ and no assertion in } \mathcal{P} \text{ is entailed by } \mathcal{Q} \text{ w.r.t. } \mathcal{T}$$

Cautious IRQ-Entailment

Cautious Entailment

An ABox query $\mathcal{Q}$ is **cautiously IRQ-entailed** by $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ iff **every** optimal IRQ-repair $\exists Y.\mathcal{B}$ of $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t. $\mathcal{T}$ satisfies $\exists Y.\mathcal{B} \models^{\mathcal{T}} \mathcal{Q}$.

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Note:

- Cannot be solved by a reduction to classical reasoning in $\mathcal{EL}$.
- One could compute all optimal repairs and then perform instance checking w.r.t. each of them, which would be very expensive.
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*Are there any **polynomial-size representations** of canonical repairs such that reasoning with them is **tractable**?*

Repair Seed Functions

Repair Seed Function (rsf)

- It assigns to each individual b a set of **atoms that should not hold for** b in the repair.
- each seed function s induces a canonical repair denoted by $\mathrm{rep}(\exists X.\mathcal{A}, s)$ **[CADE'21]**

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Reasoning w.r.t. Canonical IQ-repairs

Given an rsf s and an ABox $\mathcal{Q}$, we can decide **in polynomial time** whether $\mathrm{rep}(\exists X.\mathcal{A}, s) \models^{\mathcal{T}} \mathcal{Q}$ **without computing** $\mathrm{rep}(\exists X.\mathcal{A}, s)$.

What seed functions that induce optimal IRQ-*repairs?*

$\leq_{\mathsf{IRQ}}$-Minimality of Seed Functions

Introducing a **pre-order relation** $\leq_{\mathsf{IRQ}}$ on seed functions that reflects IRQ-entailment between canonical repairs.

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s is an $\leq_{\mathsf{IRQ}}$-**minimal** seed function iff $\mathsf{rep}(\exists Z.\mathcal{C}, s)$ is an optimal IRQ-repair.

Complexity of the $\leq_{\mathsf{IRQ}}$-Minimality Problem

Deciding whether a seed function is $\leq_{\mathsf{IRQ}}$-minimal can be done in polynomial time.

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Cautious Entailment is in coNP (*unclear if it is coNP-hard*)

Non-Cautious Entailment: guess a function s mapping each individual to a set of atoms in the ontology and then check whether

- s is a repair seed function and is $\leq_{\text{IRQ}}$-minimal, and
- $\text{rep}(\exists Z.\mathcal{C}, s) \not\models^{\mathcal{T}} \mathcal{Q}$.

Classical Repairs of qABoxes w.r.t. $\mathcal{EL}$ TBoxes

If we consider classical repairs of qABoxes, then we obtain a matching lower bound for brave and cautious reasoning.

Brave Entailment is NP-Complete

- Guessing a subset of $\mathcal{A}$ and check whether it entails $\mathcal{Q}$ w.r.t. $\mathcal{T}$ and is maximal.
- Obtaining NP-hardness by a reduction from the problem **monotone 1-in-3-SAT**.

Cautious Entailment is coNP-Complete

Considering Non-Cautious Entailment

- Guessing a subset of $\mathcal{A}$ and check whether it does not entail $\mathcal{Q}$ w.r.t. $\mathcal{T}$ and is maximal.
- Obtaining NP-hardness by a reduction from the problem **path via a node**.

Conclusions and Future Work

Conclusion:

- Computing optimal IRQ-repairs for ABox repair request can be performed in exponential time.
- Investigated the complexities of brave and cautious entailment of qABoxes based on optimal and classical repairs.
- Characterized the $\leq_{\mathsf{IRQ}}$-minimality of seed functions.

Future Work:

- Is CoNP upper bound for cautious IRQ-entailment really **tight**?
- Considering **more variants of repairs** for error-tolerant reasoning
- **Inconsistent-tolerant** reasoning