TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science • Institute of Theoretical Computer Science • Chair of Automata Theory

Pushing Optimal ABox Repair From $\mathcal{EL}$ Towards More Expressive Horn-DLs

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- **Classical Approach:** Delete a **minimal number of axioms** from the ontology such that the unwanted consequences vanish.
- This is easy to understand and implement, but might destroy too many other consequences.
- **Novel Approach:** Modify the ontology such that a **minimal number of consequences** is removed, including the unwanted ones.

“Optimal Repair”

Basics

The Description Logic $\mathcal{ELROI}$

- Signature Σ consisting of individual names a, concept names A, and role names r
- Roles $R ::= r \mid r^-$
- Concept descriptions $C ::= \top \mid A \mid \{a\} \mid C \sqcap C \mid \exists R.C$

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- TBox $\mathcal{T}$: finite set of concept inclusions $C \sqsubseteq D$
- RBox $\mathcal{R}$: finite set of role inclusions $R_1 \circ \cdots \circ R_n \sqsubseteq S$

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Quantified ABoxes with Equalities

- Quantified ABox $\exists X.\mathcal{A}$: finite set X of variables, and finite set $\mathcal{A}$ of (flat) concept assertions $A(u)$, role assertions $r(u, v)$, and equality assertions $a \equiv b$, where u and v are individual names or variables.
- Entailment: $\exists X.\mathcal{A} \models^{\mathcal{T},\mathcal{R}} \exists Y.\mathcal{B}$ if each model of $\exists X.\mathcal{A}$ and $(\mathcal{T}, \mathcal{R})$ is a model of $\exists Y.\mathcal{B}$.

Repairs

Basic Definitions

- A **repair request** $\mathcal{P}$ is a union of
  1. the local request $\mathcal{P}_{\text{loc}}$: finite set of concept assertions $C(a)$
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- $\exists Y.\mathcal{B}$ is a **repair** of $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t. $(\mathcal{T},\mathcal{R})$ if
  1. $\exists X.\mathcal{A} \models^{\mathcal{T},\mathcal{R}} \exists Y.\mathcal{B}$
  2. $\exists Y.\mathcal{B} \not\models^{\mathcal{T},\mathcal{R}} C(a)$ for each $C(a) \in \mathcal{P}_{\text{loc}}$
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- A repair $\exists Y.\mathcal{B}$ is **optimal** if it is not strictly entailed by another repair,
  i.e., there is no other repair $\exists Z.\mathcal{C}$ such that $\exists Z.\mathcal{C} \models^{\mathcal{T},\mathcal{R}} \exists Y.\mathcal{B}$ but $\exists Y.\mathcal{B} \not\models^{\mathcal{T},\mathcal{R}} \exists Z.\mathcal{C}$.

Basics

Entailment of Quantified ABoxes

Assume that $(\mathcal{T}, \mathcal{R})$ is an $\mathcal{ELROI}$ terminology.

If $\exists Y.\mathcal{B}$ can be constructed from $\exists X.\mathcal{A}$ by a finite number of applications of the following rules, then $\exists X.\mathcal{A} \models^{\mathcal{T},\mathcal{R}} \exists Y.\mathcal{B}$.

Delete Rule: Delete an assertion from $\exists X.\mathcal{A}$.

Copy Rule: Copy an object in $\exists X.\mathcal{A}$ into a fresh variable.

CI Rule: If $C \sqsubseteq D \in \mathcal{T}$ and $\mathcal{A} \models C(t)$ but $\mathcal{A} \not\models D(t)$, then unfold $D(t)$ into $\exists X.\mathcal{A}$.

RI Rule: If $R_1 \circ \cdots \circ R_n \sqsubseteq S \in \mathcal{R}$ and $\mathcal{A} \models R_1(t_0, t_1), \ldots, R_n(t_{n-1}, t_n)$ but $\mathcal{A} \not\models S(t_0, t_n)$, then add $S(t_0, t_n)$ to $\exists X.\mathcal{A}$.

Terminating Terminologies

Example

- qABox: $\exists\emptyset.\{A(a)\}$
- TBox: $\{A \sqsubseteq \exists r.A,\ \exists r.A \sqsubseteq A\}$
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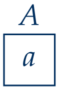

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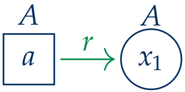

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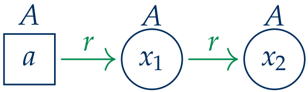

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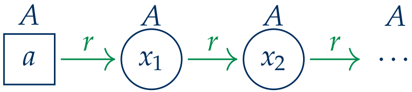

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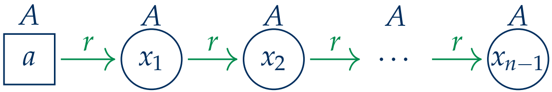

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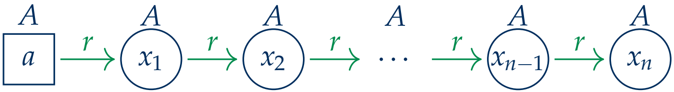

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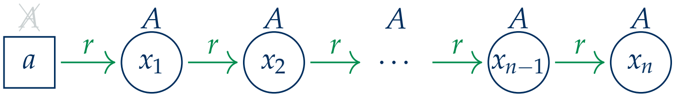

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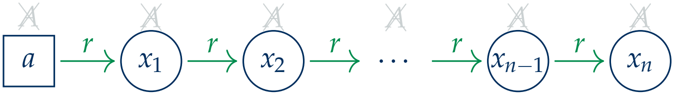

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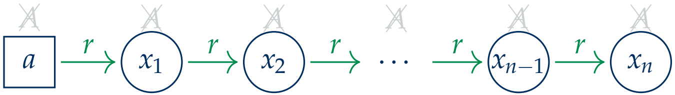

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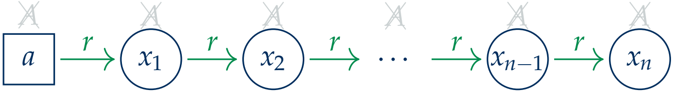

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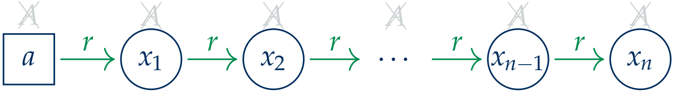

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- Then every qABox $\exists X.\mathcal{A}$ has a **saturation** $\mathsf{sat}^{\mathcal{T},\mathcal{R}}(\exists X.\mathcal{A})$ that is obtained by exhaustively applying the CI Rule and the RI Rule to $\exists X.\mathcal{A}$.
- $\exists X.\mathcal{A} \models^{\mathcal{T},\mathcal{R}} \exists Y.\mathcal{B}$ if and only if there is a homomorphism from $\exists Y.\mathcal{B}$ to $\mathsf{sat}^{\mathcal{T},\mathcal{R}}(\exists X.\mathcal{A})$ (and also: if and only if $\exists Y.\mathcal{B}$ can be constructed from $\exists X.\mathcal{A}$ by means of the four rules).

Regular RBoxes

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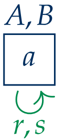

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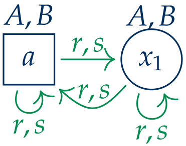

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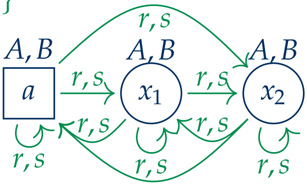

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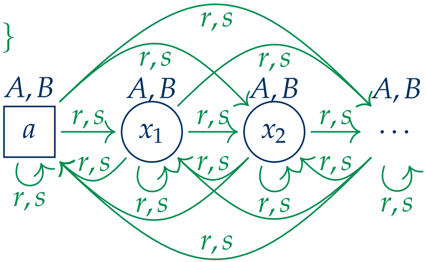

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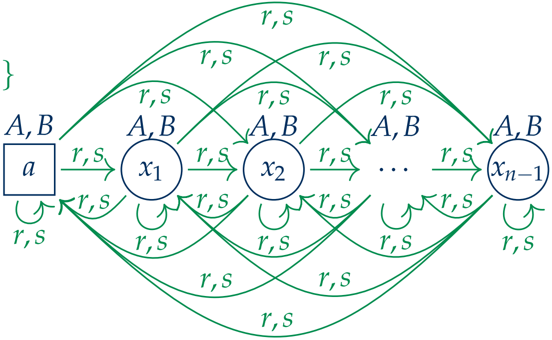

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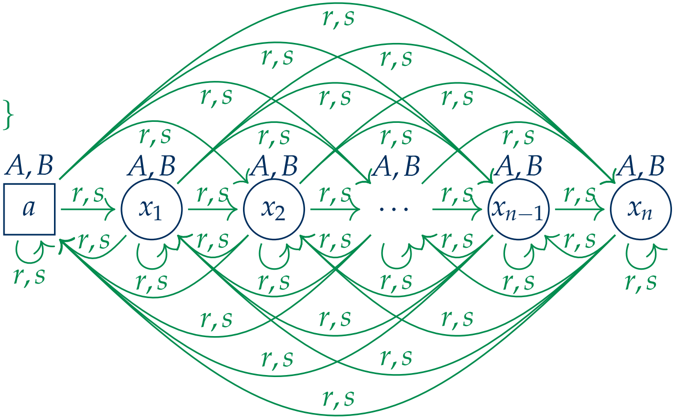

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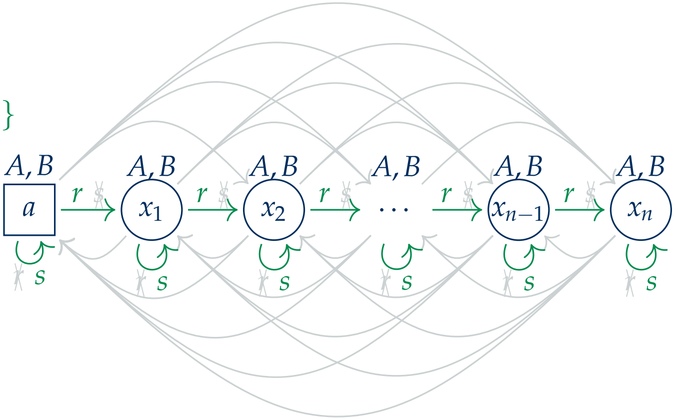

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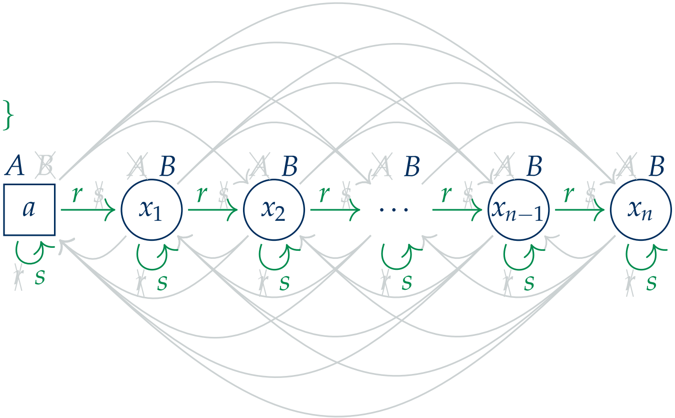

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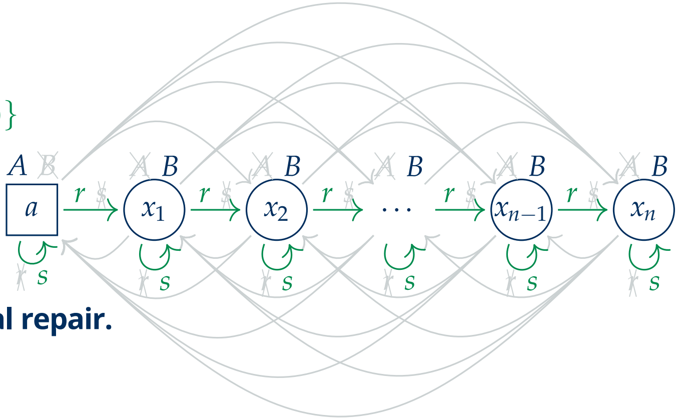

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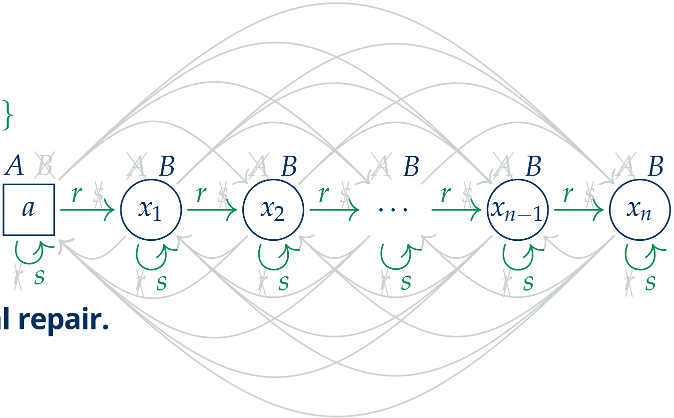

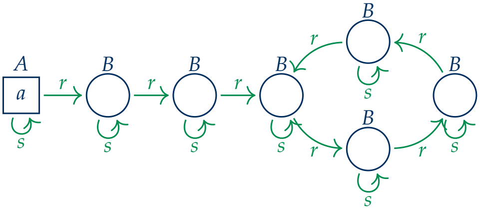

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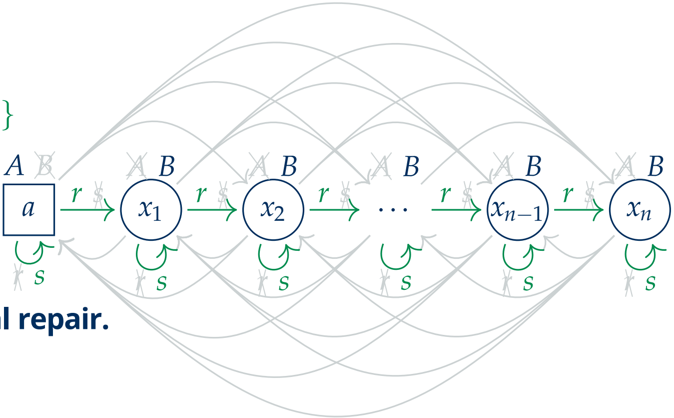

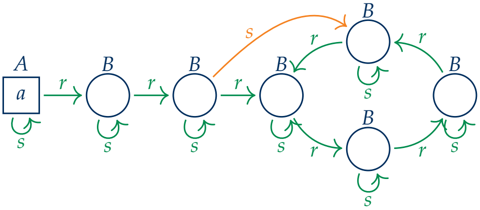

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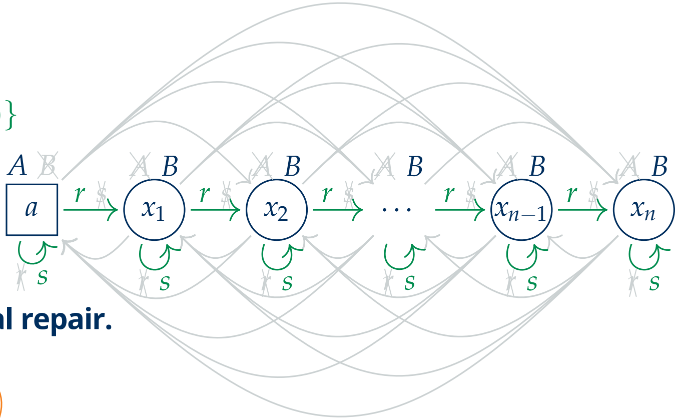

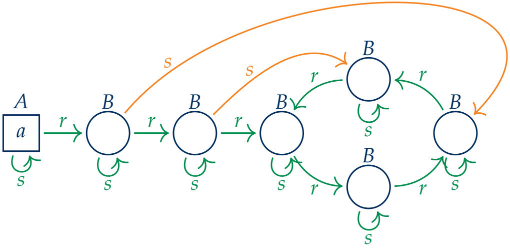

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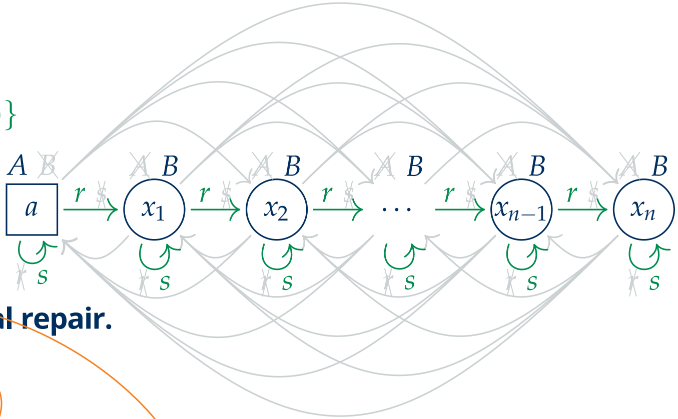

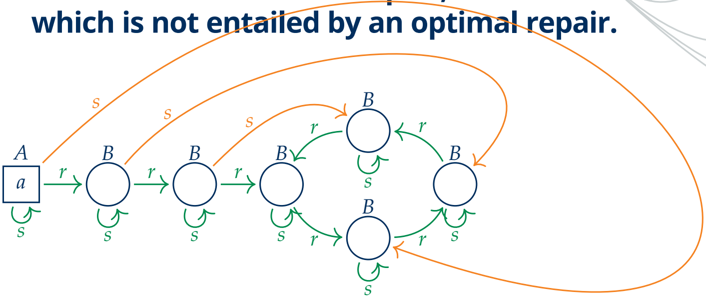

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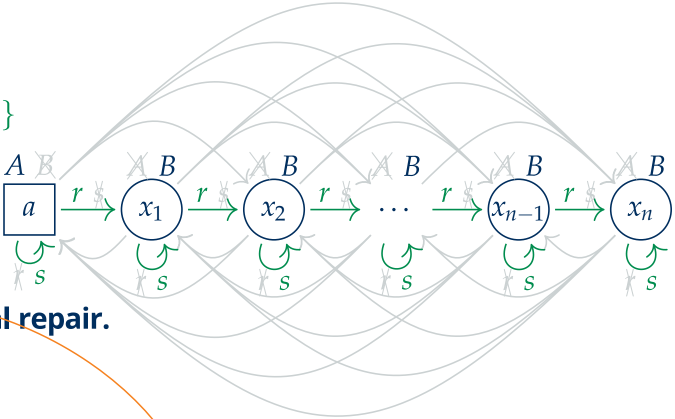

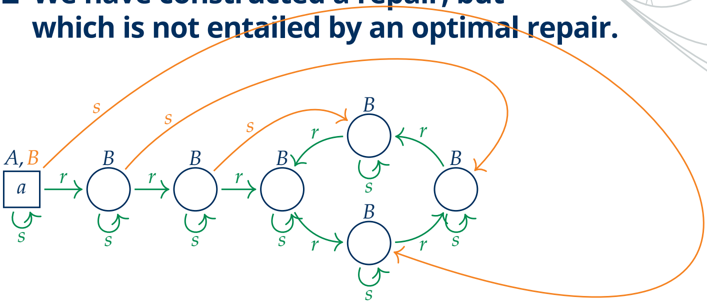

Regular RBoxes

Example

- qABox: $\exists\emptyset.\{r(a,a),\ s(a,a),\ A(a),\ B(a)\}$
- TBox: $\{\exists s.A \sqsubseteq A,\ \exists s.B \sqsubseteq B\}$
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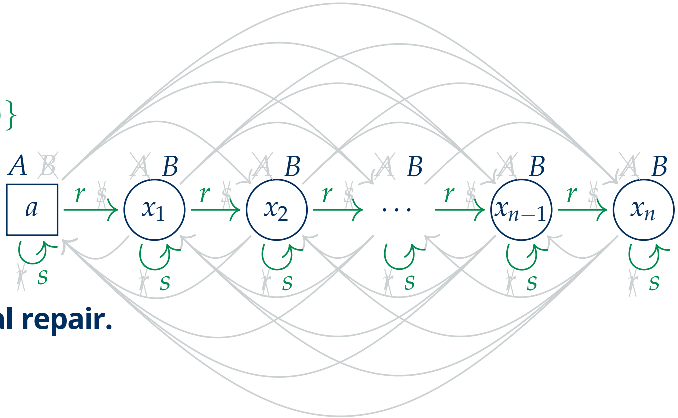

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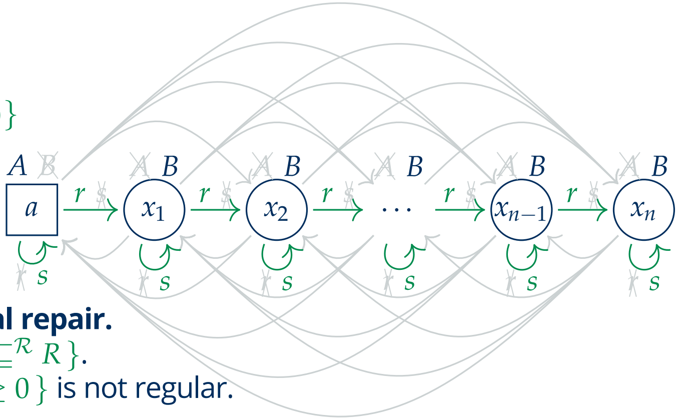

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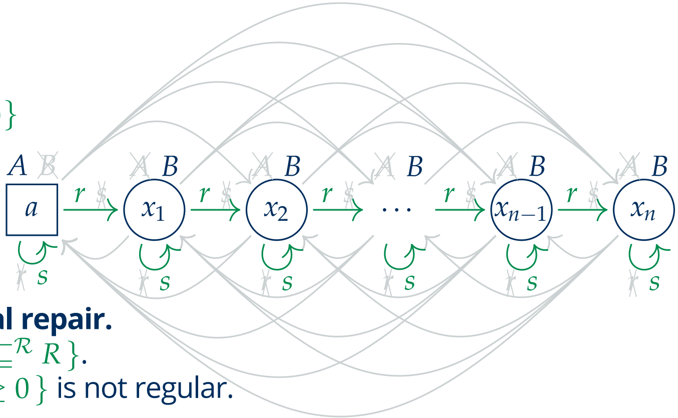

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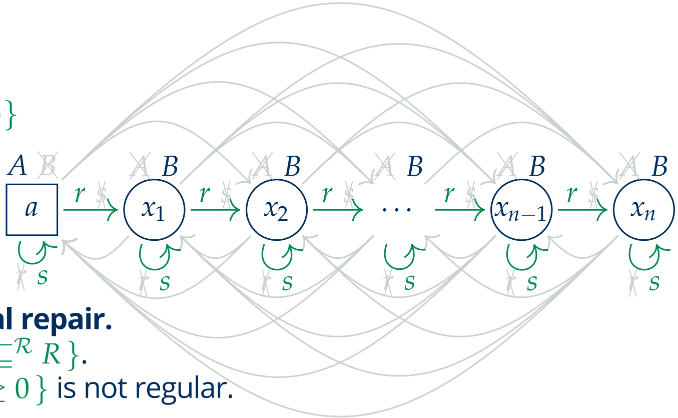

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- Then every language $L_{\mathcal{R}}(R)$ is accepted by some finite automaton $\mathfrak{A}_R$.
- We introduce an **automaton concept** $\exists q.C$ for each atom $\exists R.C$ occurring in $\mathcal{P}$ or $\mathcal{T}$ and for each state q in $\mathfrak{A}_R$.
- $\exists q.C$ represents the disjunction $\bigsqcup\{\, \exists S_1. \cdots \exists S_n.C \mid \mathfrak{A}_R$ with initial state q accepts $S_1 \cdots S_n \,\}$.

Small Repair Property

- By means of filtration, we can show a **small repair property**.
- Every repair of $\exists X.\mathcal{A}$ for $\mathcal{P}$ w.r.t. $(\mathcal{T},\mathcal{R})$ is entailed by a repair with at most $m \cdot 2^n$ objects, where
  - m is the number of objects in $\mathsf{sat}^{\mathcal{T},\mathcal{R}}(\exists X.\mathcal{A})$
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- **Theorem:** 1 **The set of all optimal repairs can effectively be computed.**
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Canonical Repairs

- Repairs can also be directly constructed from the input.
- A **repair type** $\mathcal{K}$ is a set consisting of atoms A and $\{a\}$ and of automaton concepts $\exists q.C$.
- $\mathcal{K}$ must be closed under premises:
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u is an object of $\exists Y.\mathcal{B}$

$\mathcal{K}$ is a repair type

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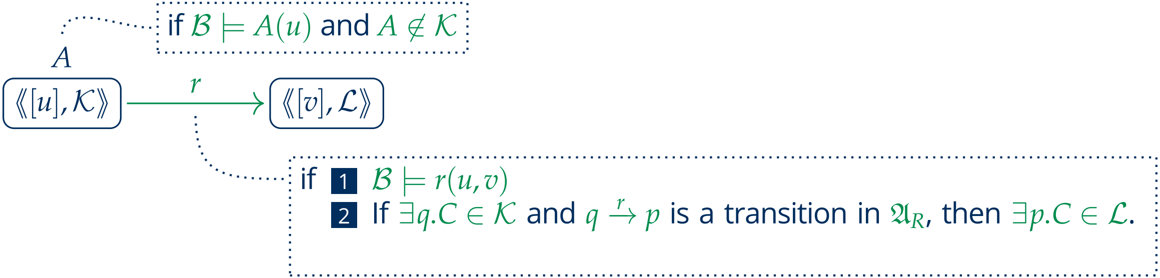

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if
1. $\mathcal{B} \models r(u,v)$
2. If $\exists q.C \in \mathcal{K}$ and $q \xrightarrow{r} p$ is a transition in $\mathfrak{A}_R$, then $\exists p.C \in \mathcal{L}$.
3. If $\exists q.C \in \mathcal{L}$ and $q \xrightarrow{r^-} p$ is a transition in $\mathfrak{A}_R$, then $\exists p.C \in \mathcal{K}$.

Canonical Repairs

- In order to fulfill the repair request $\mathcal{P}$,
  1. each individual name a must be represented by a copy $\langle\!\langle [a], \mathcal{K} \rangle\!\rangle$ where $\mathsf{Conj}_{\mathcal{R}}(C) \cap \mathcal{K} \neq \emptyset$ for each $C(a) \in \mathcal{P}_{\mathsf{loc}}$.
  2. and the canonical repair only contains copies $\langle\!\langle [u], \mathcal{K} \rangle\!\rangle$ where $\mathsf{Conj}_{\mathcal{R}}(D) \cap \mathcal{K} \neq \emptyset$ for each $D \in \mathcal{P}_{\mathsf{glo}}$.
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- **Completeness:** Given an arbitrary repair, the mapping $u \mapsto \langle\!\langle [h(u)], \mathcal{F}(u) \rangle\!\rangle$ is a homomorphism to a canonical repair, where
  - h is a homomorphism to the saturation $\mathsf{sat}^{\mathcal{T},\mathcal{R}}(\exists X.\mathcal{A})$
  - and the repair type $\mathcal{F}(u)$ consists of all atoms and automaton concepts that are not satisfied by u.
- **Theorem:** 1. **Each canonical repair is a repair and can effectively be computed.**
  2. **Every repair is entailed by a canonical repair.**
- **Corollary: Each optimal repair is equivalent to a canonical repair.**

User Interaction

- Usually, there is no unique optimal repair.
- Exponentially many optimal repairs can exist in the worst case. The source of non-determinism are conjunctions that either are in $\mathcal{P}$ or imply a concept in $\mathcal{P}$.
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- **If no question has been ignored, then there is a unique optimal repair** that entails all accepted assertions but does not entail any rejected assertion or any unwanted consequence in $\mathcal{P}$.

Inconsistency Repairs

- We extend $\mathcal{ELROI}$ with the **bottom concept** $\bot$.
- A qABox can now be inconsistent w.r.t. a terminology, e.g., $\{A(a)\}$ w.r.t. $\{A \sqsubseteq \bot\}$.
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- In order to repair for inconsistency, we add to the global request $\mathcal{P}_{\mathsf{glo}}$ every concept C where $C \sqsubseteq \bot$ is in $\mathcal{T}_{\bot}$.
- Since everything is entailed by a qABox $\exists X.\mathcal{A}$ that is inconsistent w.r.t. $(\mathcal{T},\mathcal{R})$, we now only require that a repair is entailed by it w.r.t. $(\mathcal{T}_{+},\mathcal{R})$.
- We have so obtained a reduction to our canonical repairs.

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- Thus, Horn-$\mathcal{SROIQ}$ cannot be fully supported if the goal is computing optimal repairs.

Do you have questions or comments?