

Query Rewriting for *DL-Lite* with n -ary Concrete Domains

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Montpellier, 19/07/2017

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TECHNISCHE
UNIVERSITÄT
DRESDEN

2

TNG

TECHNOLOGY
CONSULTING



Measurements

Patient	Heart rate	Age	Timestamp
p78435	80	45	2017-06-09T11:46:50
p71247	60	60	2017-06-09T11:47:05
p72349	120	85	2017-06-09T11:48:19
p73872	160	80	2017-06-09T12:23:16
⋮	⋮	⋮	⋮
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⋮	⋮	⋮	⋮
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Select Patient From Measurements Where HR > 140



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Select Patient From Measurements Where **HR > 140**

$$\exists h. \text{Patient}(p) \wedge \text{HR}(p, h) \wedge \textcolor{red}{h} > \textcolor{red}{140}$$



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Select Patient From Measurements Where **HR > 140**

$$\exists h. \text{Patient}(p) \wedge \text{HR}(p, h) \wedge \mathbf{h > 140}$$

$$\text{MaxHR} = 220 - \text{Age} \quad \text{HR} = \text{MaxHR}: \text{Critical!}$$



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Patient(p) $\wedge$ **Critical(p)**

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Select Patient From Measurements Where **HR = 220 - Age**

Patient(p) $\wedge$ **Critical(p)**

rewritable?

MaxHR = 220 - Age

HR = MaxHR: Critical!

DL-Lite with n -ary Concrete Domains

Concrete domain $\mathcal{D}$

domain $\Delta^{\mathcal{D}}$

predicates $P_i^{\mathcal{D}} \subseteq (\Delta^{\mathcal{D}})^{n_i}$

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$DL\text{-}Lite_{horn}^{(\mathcal{H}\mathcal{F})}(\mathcal{D})$

Patient, Doctor, Critical, ...

(atomic concepts)

HasDoctor, ...

(roles)

p73872, p78435, ...

(individuals)

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HR, Age, ...

(attributes)

160, 220, ...

(constants)

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160, 220, ...

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ABox $\mathcal{A}$: Patient(p73872) HR(p73872, 160) Age(p73872, 80)

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TBox $\mathcal{T}$: Patient $\sqsubseteq \exists \text{MaxHR}$ Patient $\sqsubseteq \forall \text{HR}, \text{HR.Eq}$

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Patient $\sqsubseteq \forall \text{MaxHR}, \text{Age.P}_{220-}$ $\exists \text{HR}, \text{MaxHR.Eq} \sqsubseteq \text{Critical}$

Concrete domain $\mathcal{D}$ ABox $\mathcal{A}$ TBox $\mathcal{T}$

Concrete domain $\mathcal{D}$ ABox $\mathcal{A}$ TBox $\mathcal{T}$ Query ϕ

$\exists h, m. \text{Patient}(p) \wedge \text{HR}(p, h) \wedge \text{MaxHR}(p, m) \wedge \text{Eq}(h, m)$



Concrete domain $\mathcal{D}$ ABox $\mathcal{A}$ TBox $\mathcal{T}$ Query ϕ

$\exists h, m. \text{Patient}(p) \wedge \text{HR}(p, h) \wedge \text{MaxHR}(p, m) \wedge h = m$



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$\text{cert}(\mathcal{A}, \mathcal{T}, \phi) = ?$

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Rewritability

Data complexity

first-order rewritable: $\text{cert}(\mathcal{A}, \mathcal{T}, \phi) = \text{ans}(\mathcal{A}, \phi_{\mathcal{T}})$

AC^0

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P

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P



does not need to be polynomial

Conditions for Rewritability

(Artale, Ryzhikov, and Kontchakov 2012; Savković and Calvanese 2012)
unary predicates, first-order rewritability

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(infinitediff)

Let ϕ be a $\mathcal{D}$ -conjunction
and $\psi = \psi_1 \vee \dots \vee \psi_n$ a $\mathcal{D}$ -disjunction.

$$x = 220 - y \wedge z = x$$

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If ϕ is satisfiable and $\text{sol}(\phi) \not\subseteq \text{sol}(\psi_j)$ for all j , then $|\text{sol}(\phi) \setminus \text{sol}(\psi)| = \infty$.

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(Baader, Brandt, and Lutz 2005)
 n -ary predicates, subsumption in $\mathcal{EL}^{++}$



Convexity

If ϕ implies ψ (i.e. $\text{sol}(\phi) \subseteq \text{sol}(\psi)$), then it implies some ψ_j .

(Artale, Ryzhikov, and Kontchakov 2012; Savković and Calvanese 2012)
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Convexity and (*infinitediff*) are equivalent
in the presence of the unary predicates $(\text{Eq}_d)^{\mathcal{D}} := \{d\}$.

Convex concrete domains

$\mathbb{Q}$ with $\mathbf{Eq}$, $\mathbf{Eq}_q$, $\mathbf{P}_{q-}$
 $\mathbb{Q}$ with $\mathbf{Eq}$, $\mathbf{Eq}_q$, $\mathbf{P}_{q+}$, $>_q$ (or $<_q$)
 Σ^* with $\mathbf{Eq}$, $\mathbf{Eq}_w$, $\mathbf{concat}_w$

Convex concrete domains

$\mathbb{Q}$ with Eq , Eq_q , P_{q-}
 $\mathbb{Q}$ with Eq , Eq_q , P_{q+} , $>_q$ (or $<_q$)
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cr-admissibility

(1) **equality** predicates Eq , Eq_d

$x = y$ $x = 100$

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cr-admissibility

- (1) equality predicates $\mathbf{Eq}$, $\mathbf{Eq}_d$ $x = y$ $x = 100$
- (2) **decidable** in polynomial time $x = 220 - y \wedge y > 50 \rightarrow x < 100 ?$

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- (1) equality predicates $\mathbf{Eq}$, $\mathbf{Eq}_d$ $x = y$ $x = 100$
- (2) decidable in polynomial time $x = 220 - y \wedge y > 50 \rightarrow x < 100 ?$
- (3) **compute** counterexamples in polynomial time $x \mapsto 160, y \mapsto 60$

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- (2) decidable in polynomial time $x = 220 - y \wedge y > 50 \rightarrow x < 100 ?$
- (3) compute counterexamples in polynomial time $x \mapsto 160, y \mapsto 60$
- (4) **functional** predicates $x = 220 - y \wedge y = 60 \rightarrow x = 160$

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- (5) **convexity**

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- (2) decidable in polynomial time $x = 220 - y \wedge y > 50 \rightarrow x < 100 ?$
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- (4) functional predicates $x = 220 - y \wedge y = 60 \rightarrow x = 160$
- (5) convexity

Safe CQs

Every concrete variable v must occur in an atom $\mathbf{U}(x, v)$ or $v = d$.

The Rewriting

Showing rewritability

Find a **canonical model** $\mathcal{I}_{\mathcal{A}, \mathcal{T}}$ such that, **for all CQs** ϕ ,

$$\text{cert}(\mathcal{A}, \mathcal{T}, \phi) = \text{ans}(\mathcal{I}_{\mathcal{A}, \mathcal{T}}, \phi) = \text{ans}(\mathcal{A}_{\mathcal{T}}, \phi_{\mathcal{T}}).$$

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$$\text{Patient}(\text{p73872}) \quad \text{Patient} \sqsubseteq \exists \text{MaxHR}.>0$$

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$$\begin{aligned} & \text{Patient}(\text{p73872}) \quad \text{Patient} \sqsubseteq \exists \text{MaxHR}.\textcolor{teal}{>0} \\ \leadsto & \text{MaxHR}(\text{p73872}, \textcolor{teal}{q}) \text{ holds in } \mathcal{I}_{\mathcal{A},\mathcal{T}} \text{ for some } \textcolor{teal}{q} > 0 \end{aligned}$$

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Patient(p73872) Patient $\sqsubseteq \exists \text{MaxHR}.>0$

$\leadsto \text{MaxHR}(\text{p73872}, q)$ holds in $\mathcal{I}_{\mathcal{A}, \mathcal{T}}$ for some $q > 0$

$\leadsto \mathcal{I}_{\mathcal{A}, \mathcal{T}}$ satisfies $\exists v. \text{MaxHR}(\text{p73872}, v) \wedge v > \frac{q}{2}$ ⚡

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Two solutions

1. Construct $\mathcal{I}_{\mathcal{A},\mathcal{T},\phi}$ by taking ϕ into account.

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Two solutions

1. Construct $\mathcal{I}_{\mathcal{A},\mathcal{T},\phi}$ by taking ϕ into account.
2. Restrict CQs to finitely many **concrete predicates**.

(Artale, Ryzhikov, and Kontchakov 2012)

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Assuming that we have $\mathcal{I}_{\mathcal{A}, \mathcal{T}}$ and $\mathcal{A}_{\mathcal{T}}$, how do we compute $\phi_{\mathcal{T}}$?

Implications in the TBox

$\text{Patient}(p) \wedge \text{Critical}(p)$

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 $\quad \quad \quad \leftarrow$

$\exists h, m. \text{Patient}(p) \wedge \text{HR}(p, h) \wedge \text{MaxHR}(p, m) \wedge h = m$

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$\leftarrow$

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Implications in the concrete domain

$$\exists m. \text{Patient}(p) \wedge \text{MaxHR}(p, m) \wedge m = 160$$

$$m = 220 - a \wedge a = 60 \rightarrow m = 160$$

Implications in the TBox

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$\leftarrow$

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Bound the number of variables!

Summary

Combined rewriting for cr-admissible concrete domains

First-order rewriting for unary predicates

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Open problems

First-order rewriting?

Polynomial query rewriting $\phi_{\mathcal{T}}$?

Query answering over concrete domains in other logics?

(Hernich, Lemos, and Wolter 2017)

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Combined rewriting for cr-admissible concrete domains

First-order rewriting for unary predicates

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Polynomial query rewriting $\phi_{\mathcal{T}}$?

Query answering over concrete domains in other logics?

(Hernich, Lemos, and Wolter 2017)

Thank you!

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