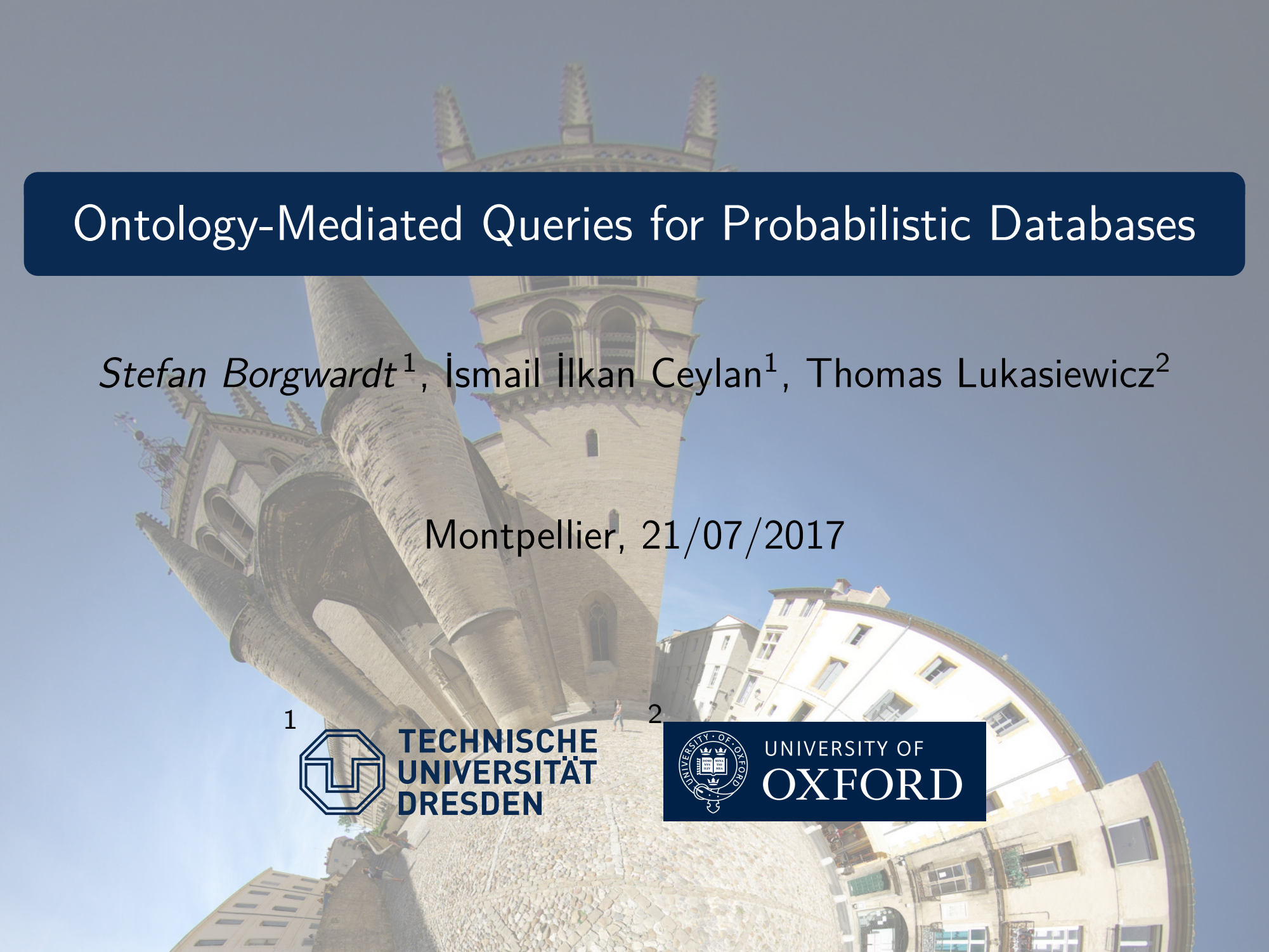
Ontology-Mediated Queries for Probabilistic Databases

Stefan Borgwardt [1], İsmail İlkan Ceylan[1], Thomas Lukasiewicz[2]

Montpellier, 21/07/2017



1 TECHNISCHE UNIVERSITÄT DRESDEN

2 UNIVERSITY OF OXFORD

Google's Knowledge Vault

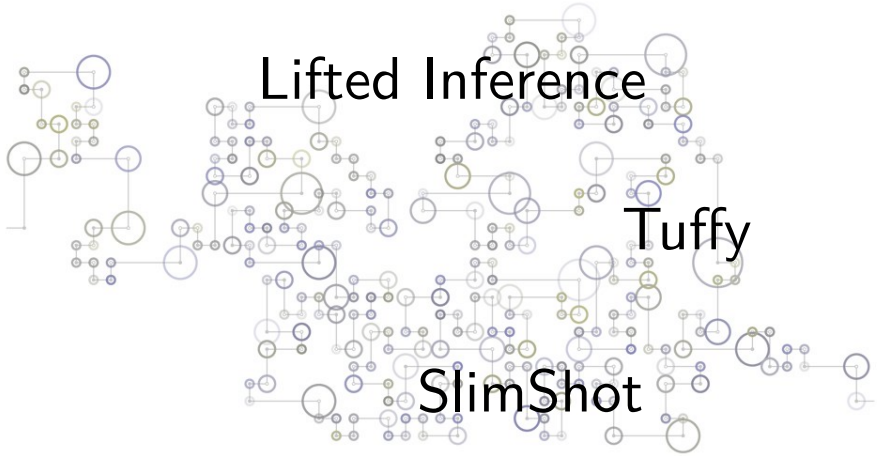
NELL

DeepDive

Google's Knowledge Vault

NELL

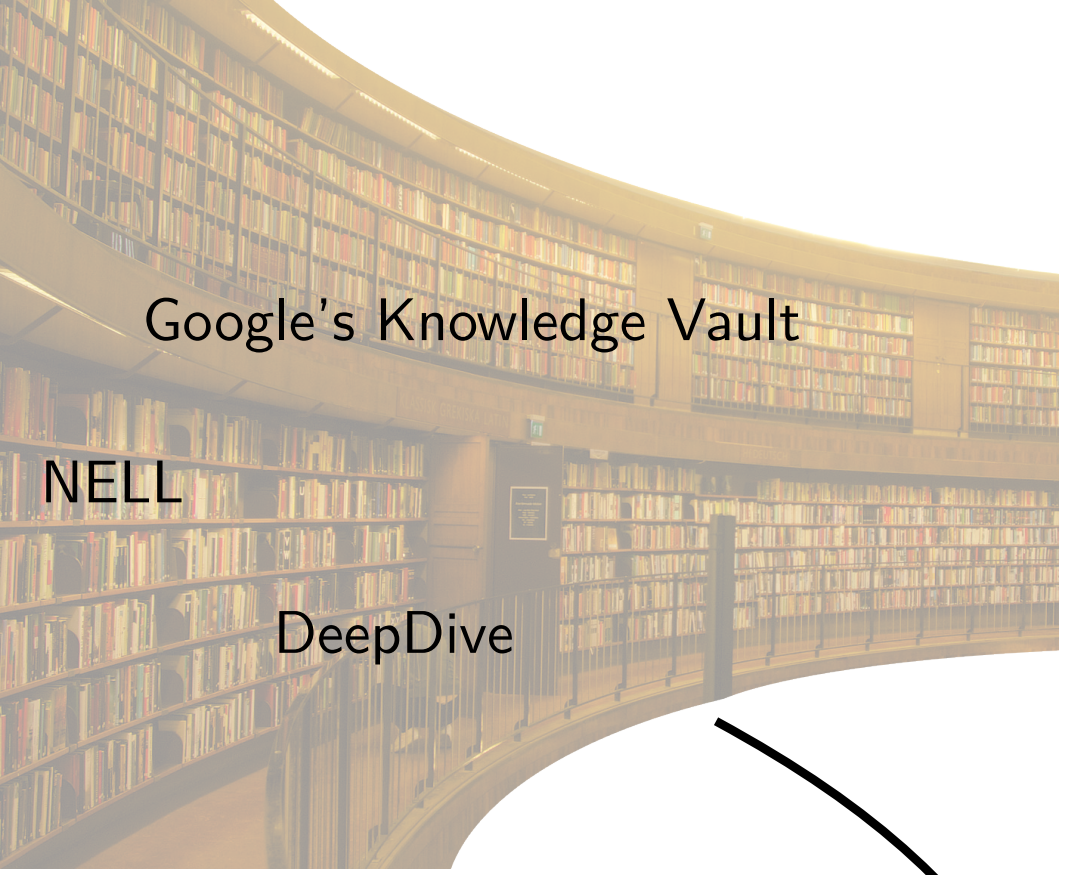
DeepDive



Lifted Inference

Tuffy

SlimShot



Lifted Inference

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Probabilistic Databases (PDBs)

Writer

Banks	0.8
Hamilton	0.7
⋮	⋮

AuthorOf

Banks	Excession	0.9
Banks	Inversions	0.8
Stephenson	Anathem	0.9
⋮	⋮	

Novel

Anathem	1
Matter	0.7
⋮	⋮

Writer	
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⋮	⋮

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⋮	⋮	

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Possible worlds $\mathcal{D}$: $\mathrm{Writer}(\mathrm{Banks})$, ~~$\mathrm{AuthorOf}(\mathrm{Banks}, \mathrm{Excession})$~~, ...

Finite

Writer	
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Hamilton	0.7
⋮	⋮

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Possible worlds $\mathcal{D}$: $\mathrm{Writer}(\mathrm{Banks})$, ~~$\mathrm{AuthorOf}(\mathrm{Banks}, \mathrm{Excession})$~~, ...

Finite

Probability $\mathrm{P}(\mathcal{D})$: $0.8 \cdot (1 - 0.9) \cdot \ldots$

Tuple-independent

Writer

Banks	0.8
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Probabilistic query evaluation

$$\mathrm{Q}: \exists x.\, \mathrm{AuthorOf}(\mathrm{Banks}, x)$$

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$$\mathsf{Q}: \ \exists x.\, \mathrm{AuthorOf}(\mathrm{Banks}, x)$$

$$\mathrm{P}(\mathsf{Q}) = \sum_{\mathcal{D} \models \mathsf{Q}} \mathrm{P}(\mathcal{D})$$

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$$\mathrm{P}(\mathsf{Q}) = \sum_{\mathcal{D} \models \mathsf{Q}} \mathrm{P}(\mathcal{D}) = 0.8 \cdot (1 - 0.9) \cdot 0.8 \cdot \ldots + \ldots = 0.98$$

$$\mathrm{P}(\exists x.\, \mathrm{AuthorOf}(\mathrm{Banks}, x) \wedge \mathrm{Novel}(x)) = 0$$

Closed-world

Open-World Probabilistic Databases (OpenPDBs)

(Ceylan, Darwiche, and Van den Broeck 2016)

$$\text{OpenPDB} = \text{PDB} + \lambda$$

$$\mathrm{P}\big(\mathbf{Novel}\big(\mathbf{Excession}\big)\big) \in [0, \lambda]$$

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$$\mathrm{P}(\mathbf{Novel}(\mathbf{Excession})) \in [0, \lambda]$$

$\rightsquigarrow$ credal set of probability distributions P

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$$\mathrm{P}(\mathbf{Novel}(\mathbf{Excession})) \in [0, \lambda]$$

$\rightsquigarrow$ credal set of probability distributions P

Lower and upper probability

$$\underline{\mathrm{P}}(\mathrm{Q}) = \min_{\mathrm{P}} \mathrm{P}(\mathrm{Q}) \qquad \overline{\mathrm{P}}(\mathrm{Q}) = \max_{\mathrm{P}} \mathrm{P}(\mathrm{Q})$$

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$$\overline{\mathrm{P}}(\exists x.\, \mathbf{AuthorOf}(\mathbf{Banks}, x) \wedge \mathbf{Novel}(x)) = 0.8372$$

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Novel	
Anathem	1
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Excession	$[0, 0.7]$
Inversions	$[0, 0.7]$

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Data complexity dichotomy

Deciding $\overline{\mathrm{P}}(\mathrm{Q}) > p$ is either in P ("safe") or PP-complete ("unsafe").

(Ceylan, Darwiche, and Van den Broeck 2016; Dalvi and Suciu 2012)

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Hamilton	0.7
Stephenson	$[0, 0.7]$
Anathem	$[0, 0.7]$

AuthorOf		
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$\mathrm{Writer}(\mathrm{Stephenson})$

$[0, 0.7]$

$\mathrm{Writer}(\mathrm{Anathem})$

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$$\mathrm{Writer}(\mathrm{Stephenson})$$

$$[0, 0.7]$$

$$\mathrm{AuthorOf}(x, y) \wedge \mathrm{Novel}(y) \rightarrow \mathrm{Writer}(x)$$

$$\mathrm{Writer}(\mathrm{Anathem})$$

$$[0, 0.7]$$

$$\mathrm{Writer}(x) \wedge \mathrm{Novel}(x) \rightarrow \bot$$

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Stephenson	$[0, 0.7]$
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Ontology-Mediated Queries (OMQs)

(UCQ Q , Datalog$^{\pm}$ program Σ)

$$\mathbf{AuthorOf}(x, y) \wedge \mathbf{Novel}(y) \rightarrow \mathbf{Writer}(x)$$

Semantics for positive OMQs

$$\mathrm{P}(\mathsf{Q}, \Sigma) = \sum_{\mathcal{D} \models (\mathsf{Q}, \Sigma)} \mathrm{P}(\mathcal{D})$$

$$\mathbf{AuthorOf}(x, y) \wedge \mathbf{Novel}(y) \rightarrow \mathbf{Writer}(x)$$

Semantics for positive OMQs

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$$\underline{\mathrm{P}}(\text{Writer}(\text{Stephenson})) = 0.9$$

$$\overline{\mathrm{P}}(\text{Writer}(\text{Stephenson})) = 0.9 + 0.1 \cdot 0.7 = 0.97$$

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$$\overline{\mathrm{P}}(\mathbf{Writer}(\mathbf{Stephenson})) = 0.9 + 0.1 \cdot 0.7 = 0.97$$

Dichotomy

For FO-rewritable positive OMQs (Q, Σ),
deciding $\overline{\mathrm{P}}(\mathrm{Q}, \Sigma) > p$ is either in P or PP-complete.

$$(\mathsf{Q}, \Sigma) \quad \rightsquigarrow \quad \mathsf{Q}_\Sigma \quad \rightsquigarrow \quad \overline{\mathrm{P}}(\mathsf{Q}_\Sigma)$$

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$$\mathbf{AuthorOf}(x, y) \wedge \mathbf{Novel}(y) \rightarrow \mathbf{Writer}(x)$$
$$\mathbf{CharacterIn}(x, y) \wedge \mathbf{Novel}(y) \rightarrow \mathbf{LiteraryCharacter}(x)$$

$$\exists x.\, \mathbf{Writer}(x) \wedge \mathbf{LiteraryCharacter}(x)$$

$$(\mathsf{Q}, \Sigma) \quad \rightsquigarrow \quad \mathsf{Q}_\Sigma \quad \rightsquigarrow \quad \overline{\mathrm{P}}(\mathsf{Q}_\Sigma)$$

$$\mathbf{AuthorOf}(x, y) \land \mathbf{Novel}(y) \rightarrow \mathbf{Writer}(x)$$
$$\mathbf{CharacterIn}(x, y) \land \mathbf{Novel}(y) \rightarrow \mathbf{LiteraryCharacter}(x)$$

$$\exists x.\, \mathbf{Writer}(x) \land \mathbf{LiteraryCharacter}(x)$$

$$\downarrow\!\!\!\!\!\!\sim$$

$$\exists x, y, z.\, \mathbf{AuthorOf}(x, y) \land \mathbf{Novel}(y) \land \mathbf{CharacterIn}(x, z) \land \mathbf{Novel}(z)$$

$$(\mathsf{Q}, \Sigma) \quad \rightsquigarrow \quad \mathsf{Q}_\Sigma \quad \rightsquigarrow \quad \overline{\mathrm{P}}(\mathsf{Q}_\Sigma)$$

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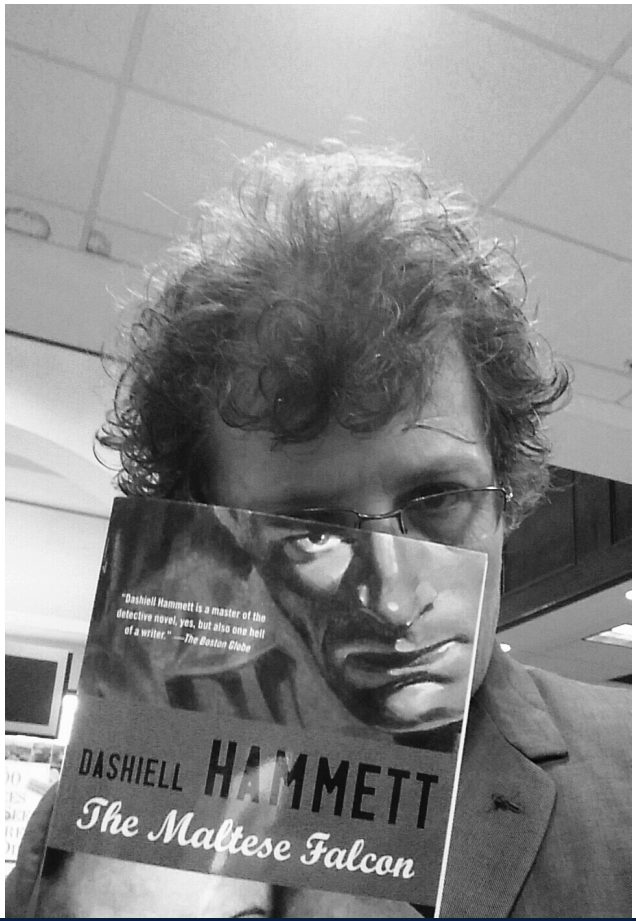
$$\mathrm{AuthorOf}(x, y) \land \mathrm{Novel}(y) \rightarrow \mathrm{Writer}(x)$$

$$\exists x, y.\, \mathrm{Writer}(x) \land \mathrm{AuthorOf}(x, y) \land \mathrm{Novel}(y)$$

$$\downarrow\!\!\!\!\!\!\sim$$

$$\exists x, y.\, \mathrm{AuthorOf}(x, y) \land \mathrm{Novel}(y)$$

$$\mathbf{Writer}(x) \wedge \mathbf{Novel}(x) \rightarrow \bot$$



$$\mathbf{Writer}(x) \wedge \mathbf{Novel}(x) \rightarrow \bot$$

Semantics with negative constraints

$$\overline{\mathrm{P}}(\mathrm{Q}, \Sigma) = \max_{\mathrm{P}} \frac{\mathrm{P}(\mathrm{Q}, \Sigma) - \mathrm{P}(\bot, \Sigma)}{1 - \mathrm{P}(\bot, \Sigma)}$$

$$\mathbf{Writer}(x) \wedge \mathbf{Novel}(x) \rightarrow \bot$$

Semantics with negative constraints

$$\overline{\mathrm{P}}(\mathrm{Q}, \Sigma) = \max_{\mathrm{P}}\{\mathrm{P}(\mathrm{Q}, \Sigma) \mid \mathrm{P}_{>0} \text{ consistent with } \Sigma\}$$

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$$\underline{\mathrm{P}}(\mathrm{Writer}(\mathrm{Anathem})) = 0$$

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Hardness result

For guarded Datalog programs,
deciding $\overline{\mathrm{P}}(\mathrm{Q}, \Sigma) > p$ is $\mathrm{NP}^{\mathrm{PP}}$-complete ($\mathrm{PP}$-complete for PDBs).

$$\exists x_1, \ldots, x_\ell.\ \mathsf{C}^c y_1, \ldots, y_m.\ \phi$$

Summary

Enriched OpenPDBs with ontology-based semantics

Lifted P/PP dichotomy for FO-rewritable positive OMQs

Negative constraints make it $\mathrm{NP}^{\mathrm{PP}}$-hard for guarded Datalog

Combined complexity results: $\mathrm{PP}^{\mathrm{NP}}/\mathrm{NP}^{\mathrm{PP}}/\ldots/$2-ExpTime

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Open problems

Positive (Q, Σ) not FO-rewritable $\Rightarrow$ PP-hard? **(Jung and Lutz 2012)**

(Q, Σ) FO-rewritable $\Rightarrow$ dichotomy?

Knowledge compilation or approximation techniques for $\mathrm{NP}^{\mathrm{PP}}$-problems?

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Thank you!

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Ceylan, İsmail İlkan, Adnan Darwiche, and Guy Van den Broeck (2016). "Open-World Probabilistic Databases". In: *Proc. of the 15th Int. Conf. on Principles of Knowledge Representation and Reasoning (KR'16)*, pages 339–348.

Dalvi, Nilesh and Dan Suciu (2012). "The Dichotomy of Probabilistic Inference for Unions of Conjunctive Queries". In: *Journal of the ACM* 59.6, 30:1–30:87.

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