Query Rewriting for *DL-Lite* with *n*-ary Concrete Domains

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Measurements

Patient	Heart rate	Age	Timestamp
p78435	80	45	2017-06-09T11:46:50
p71247	60	60	2017-06-09T11:47:05
p72349	120	85	2017-06-09T11:48:19
p73872	160	80	2017-06-09T12:23:16
⋮	⋮	⋮	⋮

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`Select` **Patient** `From` **Measurements** `Where` HR > 140

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$$\exists h.\, \mathrm{Patient}(p) \wedge \mathrm{HR}(p, h) \wedge h > 140$$

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$$\exists h.\, \text{Patient}(p) \wedge \text{HR}(p, h) \wedge h > 140$$

$$\text{MaxHR} = 220 - \text{Age} \qquad \text{HR} = \text{MaxHR} \rightarrow \text{Critical!}$$

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`Select` **Patient** `From` **Measurements** `Where` HR = 220 − Age

$$\text{Patient}(p) \wedge \text{Critical}(p)$$

rewritable?

$$\text{MaxHR} = 220 - \text{Age} \qquad \text{HR} = \text{MaxHR} \rightarrow \text{Critical}!$$

DL-Lite with *n*-ary Concrete Domains

Concrete domain $\mathcal{D}$

domain $\Delta^{\mathcal{D}}$ predicates $\mathbf{P}_i^{\mathcal{D}} \subseteq (\Delta^{\mathcal{D}})^{n_i}$

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$$(\mathrm{Eq})^{\mathcal{D}} := \{(x, y) \mid x = y\}$$
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DL-Lite$_{horn}^{(\mathcal{HF})}(\mathcal{D})$

Patient, Doctor, Critical, ... (atomic concepts)

HasDoctor, ... (roles)

p73872, p78435, ... (individuals)

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160, 220, ... (constants)

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ABox $\mathcal{A}$: $\mathrm{Patient}(\mathrm{p73872})$ $\mathrm{HR}(\mathrm{p73872}, 160)$ $\mathrm{Age}(\mathrm{p73872}, 80)$

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$\mathrm{Patient} \sqsubseteq \forall \mathrm{MaxHR}, \mathrm{Age}.\mathrm{P}_{220-}$ $\exists \mathrm{HR}, \mathrm{MaxHR}.\mathrm{Eq} \sqsubseteq \mathrm{Critical}$

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$$\exists h, m.\, \mathbf{Patient}(p) \wedge \mathbf{HR}(p, h) \wedge \mathbf{MaxHR}(p, m) \wedge h = m$$

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Rewritability	Data complexity
first-order rewritable: $\mathrm{cert}(\mathcal{A}, \mathcal{T}, \phi) = \mathrm{ans}(\mathcal{A}, \phi_{\mathcal{T}})$	AC^0

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combined rewritable:	$\mathrm{cert}(\mathcal{A}, \mathcal{T}, \phi) = \mathrm{ans}(\mathcal{A}_{\mathcal{T}}, \phi_{\mathcal{T}})$	P

Conditions for Rewritability

Convex concrete domains

(Artale, Ryzhikov, and Kontchakov 2012; Baader, Brandt, and Lutz 2005; Savković and Calvanese 2012)

If $\phi_1 \wedge \cdots \wedge \phi_m$ implies $\psi_1 \vee \cdots \vee \psi_n$, then it implies some ψ_j.

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$$x = 220 - y \;\rightarrow\; x < 200 \;\vee\; y \leq 20$$

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Examples: $\mathbb{Q}$ with $\mathbf{Eq}$, $\mathbf{Eq}_q$, $\mathbf{P}_{q-}$
$\mathbb{Q}$ with $\mathbf{Eq}$, $\mathbf{Eq}_q$, $\mathbf{P}_{q+}$, $>_q$ (or $<_q$)
Σ^* with $\mathbf{Eq}$, $\mathbf{Eq}_w$, $\mathbf{concat}_w$

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cr-admissibility

(1) equality predicates $\mathbf{Eq}$, $\mathbf{Eq}_d$ $\qquad x = y \qquad x = 100$

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(1) equality predicates $\mathbf{Eq}$, $\mathbf{Eq}_d$ $\qquad x = y \qquad x = 100$

(2) decidable in polynomial time $\qquad x = 220 - y \wedge y > 50 \rightarrow x < 100$?

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(3) compute counterexamples in polynomial time $\qquad x \mapsto 160,\ y \mapsto 60$

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(4) functional predicates $\qquad x = 220 - y \wedge y = 60 \rightarrow x = 160$

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(5) convexity

The Rewriting

Implications in the TBox

$$\mathrm{Patient}(p) \wedge \mathrm{Critical}(p)$$

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$$\text{Patient}(p) \wedge \text{Critical}(p)$$

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Implications in the concrete domain

$$\exists m.\, \text{Patient}(p) \wedge \text{MaxHR}(p, m) \wedge m = 160$$

$$m = 220 - a \wedge a = 60 \rightarrow m = 160$$

Implications in the TBox

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Bound the number of new atoms!

Summary

Combined rewriting for cr-admissible concrete domains

First-order rewriting for unary predicates

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Open problems

First-order rewriting?

Polynomial query rewriting $\phi_{\mathcal{T}}$?

Query answering over concrete domains in other logics?

(Hernich, Lemos, and Wolter 2017)

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Thank you!

Artale, Alessandro, Vladislav Ryzhikov, and Roman Kontchakov (2012). "*DL-Lite* with Attributes and Datatypes". In: *Proc. of the 20th Eur. Conf. on Artificial Intelligence (ECAI'12)*, pages 61–66.

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