DECIDABLE GÖDEL DESCRIPTION LOGICS
WITHOUT THE FINITELY-VALUED MODEL PROPERTY

Stefan Borgwardt    Felix Distel    Rafael Peñaloza

Wien, July 23rd, 2014
Motivation

- vagueness often modeled ad-hoc, e.g. $\exists$severity.High in SNOMED CT
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- more principled approach: fuzzy sets

\[ \text{Tall} \]

(Zadeh 1965)
Motivation

- vagueness often modeled ad-hoc, e.g. $\exists \text{severity}. \text{High}$ in SNOMED CT
- more principled approach: fuzzy sets
  
  ![Fuzzy Sets Diagram](image)

  - mathematical fuzzy logic
  - combination of vague predicates via suitable functions

Happy $\cap$ Successful

(Zadeh 1965)

(Hájek 2001)
Mathematical Fuzzy Logic

Happy \sqcap Successful
Mathematical Fuzzy Logic

Happy \( \sqcap \) Successful

- **t-norm** \( \otimes : [0, 1] \times [0, 1] \rightarrow [0, 1] \):
  associative, commutative, monotone, unit 1, (continuous)
Mathematical Fuzzy Logic

Happy $\cap$ Successful

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  associative, commutative, monotone, unit 1, (continuous)

- **residuum** $\Rightarrow: [0, 1] \times [0, 1] \rightarrow [0, 1]$:
  $(x \otimes y) \leq z$ iff $y \leq (x \Rightarrow z)$
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- **involutive negation** $1 - x$

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  \) \( (x \otimes y) \leq z \iff y \leq (x \Rightarrow z) \)
- involutive negation \( 1 - x \)

- **Gödel** \( (G) : \min\{x, y\} \)

![Diagram showing the Gödel t-norm](image_url)
Mathematical Fuzzy Logic

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- Gödel ($G$): $\min\{x, y\}$
- Product ($\Pi$): $x \cdot y$

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Mathematical Fuzzy Logic

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- **Gödel** ($G$): $\min\{x, y\}$
- **Product** ($\Pi$): $x \cdot y$
- **Łukasiewicz** ($Ł$): $\max(0, x + y - 1)$

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Decidable Gödel DLs without the FVMP
The Fuzzy DL $\mathcal{IALC}$

Fuzzy interpretations $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$:
- concept names: $\text{Happy}^\mathcal{I} : \Delta^\mathcal{I} \rightarrow [0, 1]$
- role names: $\text{hasFriend}^\mathcal{I} : \Delta^\mathcal{I} \times \Delta^\mathcal{I} \rightarrow [0, 1]$
- individual names: $\text{stefan}^\mathcal{I} \in \Delta^\mathcal{I}$
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Constructors:

- top $\top^\mathcal{I}(x) = 1$
- conjunction $(\text{Happy} \sqcap \text{Successful})^\mathcal{I}(x) = \min(\text{Happy}^\mathcal{I}(x), \text{Successful}^\mathcal{I}(x))$
The Fuzzy DL G-$\mathcal{I}ALC$

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- existential restriction
  
  $\exists\text{hasFriend}.\text{Happy}^{\mathcal{I}}(x) = \sup_{y \in \Delta^{\mathcal{I}}} \min(\text{hasFriend}^{\mathcal{I}}(x, y), \text{Happy}^{\mathcal{I}}(y))$
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  $$(\exists \text{hasFriend}. \text{Happy})^\mathcal{I}(x) = \sup_{y \in \Delta^\mathcal{I}} \min(\text{hasFriend}^\mathcal{I}(x, y), \text{Happy}^\mathcal{I}(y))$$
- $\text{Successful} \rightarrow \text{Happy, } \neg \text{Happy, } \forall \text{hasFriend}. \text{Successful}$
The Fuzzy DL G-\(\mathcal{ALC}\)

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- \(\text{Successful} \rightarrow \text{Happy}, \neg \text{Happy}, \forall \text{hasFriend}.\text{Successful}\)

Witnessed interpretations: \(^{(\text{Hájek 2005})}\)

\[ (\exists \text{hasFriend}.\text{Happy})^\mathcal{I}(x) = \max_{y \in \Delta^\mathcal{I}} \min(\text{hasFriend}^\mathcal{I}(x, y), \text{Happy}^\mathcal{I}(y)) \]
Reasoning

Axioms: order assertions

\langle \text{hasFriend: (stefan, felix) } \geq 0.7 \rangle \quad \langle \text{felix: Successful } > \text{ rafael: Successful} \rangle
Reasoning

Axioms: order assertions and general concept inclusions (GCIs)

\( \langle \text{hasFriend} : (\text{stefan, felix}) \geq 0.7 \rangle \quad \langle \text{felix: Successful} > \text{rafael: Successful} \rangle \)

\( \langle \exists \text{hasFriend.} \neg \text{Happy} \sqsubseteq \neg \text{Happy} \geq 0.6 \rangle \)
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Reasoning tasks:

- **consistency:**
  Does \( \mathcal{O} \) have a witnessed model?

- **satisfiability** to degree \( p \):
  Is there a witnessed model of \( \mathcal{O} \) such that \( C^\mathcal{I}(x) \geq p \) for some \( x \)?

- **subsumption** to degree \( p \):
  Is \( \langle C \sqsubseteq D \geq p \rangle \) satisfied by all witnessed models of \( \mathcal{O} \)?
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- What is the best satisfiability/subsumption degree?
The Finitely-Valued Model Property

FVMP: Every consistent ontology has a model that uses only finitely many values.
The Finitely-Valued Model Property

**FVMP:** Every consistent ontology has a model that uses only finitely many values.

- $\Pi$-$\mathcal{AL}$ and Ł-$\mathcal{AL}$ do not have the FVMP. (Bobillo, Bou, and Straccia 2011)

- Consistency in $\Pi$-$\mathcal{I}$-$\mathcal{AL}$ is undecidable. (Baader and Peñaloza 2011; Borgwardt and Peñaloza 2012)

- Consistency in Ł-$\mathcal{I}$-$\mathcal{AL}$ is undecidable. (Cerami and Straccia 2013)

- $\Pi$-$\mathcal{I}$-$\mathcal{AL}$ and G-$\mathcal{I}$-$\mathcal{AL}$ with only $\geq$-assertions have the FVMP and decidable consistency problems. (Borgwardt, Distel, and Peñaloza 2012)

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Gödel DLs without the FVMP
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\[\text{FVMP} \equiv \text{decidability} ?\]

\(G\mathcal{IALC}\) does not have the FVMP, but consistency is decidable.
The Finitely-Valued Model Property in G-\(\mathcal{AL}\)

\[
x \Rightarrow y = \begin{cases} 
1 & \text{if } x \leq y \\
y & \text{otherwise}
\end{cases}
\]

\[
\langle \text{felix: Happy } = 0.8 \rangle \quad \langle \forall \text{hasFriend.Happy } \sqsubseteq \text{ Happy} \rangle \quad \langle \exists \text{hasFriend.T } \sqsubseteq \text{ Happy} \rangle
\]
The Finitely-Valued Model Property in G-$\mathcal{AL}$

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Happy : 0.8
The Finitely-Valued Model Property in G-$\cal L$

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\[ \text{hasFriend}(x, y) \Rightarrow \text{Happy}(y) \leq \text{Happy}(x) \quad \text{hasFriend}(x, y) \leq \text{Happy}(x) \]

\[ \begin{align*}
\text{hasFriend: 0.8} & \\
\text{Happy: 0.8} & > \quad \text{Happy: 0.7}
\end{align*} \]
The Finitely-Valued Model Property in G-$\mathcal{AL}$

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\[
\text{Happy}: 0.8 > \text{Happy}: 0.7 > \text{Happy}: 0.65
\]
Only the Order Matters

Happy: 0.8

hasFriend: 0.8

Happy: 0.7

hasFriend: 0.7

Happy: 0.65

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Decidable Gödel DLs without the FVMP
Only the Order Matters

HasFriend: 0.8
Happy: 0.8

HasFriend: 0.7
Happy: 0.7

HasFriend: 0.65
Happy: 0.65

abstract:

Happy = 0.8

0 < Happy < hasFriend ≤ Happy↑ = 0.8

0 < Happy < hasFriend ≤ Happy↑ < 0.8
Only the Order Matters

Happy: 0.8  hasFriend: 0.8  Happy: 0.7  hasFriend: 0.7  Happy: 0.65

abstract:

Happy = 0.8  0 < Happy < hasFriend ≤ Happy↑ = 0.8  0 < Happy < hasFriend ≤ Happy↑ < 0.8

Hintikka trees consisting of Hintikka orderings:

0 < ∀hasFriend.Happy < Happy ≡ ∃hasFriend.⊤ ≡ (∀hasFriend.Happy)↑ < hasFriend ≡ A↑ ≡ (∃hasFriend.⊤)↑ < 0.2 < 0.5 < 0.8 < 1 ≡ ⊤ ≡ ⊤↑
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\[ 0 < \forall \text{hasFriend}. \text{Happy} < \text{Happy} \equiv \exists \text{hasFriend}. \top \equiv (\forall \text{hasFriend}. \text{Happy})_\uparrow < \text{hasFriend} \equiv A_\uparrow \equiv (\exists \text{hasFriend}. \top)_\uparrow < 0.2 < 0.5 < 0.8 < 1 \equiv \top \equiv \top_\uparrow \]

looping tree automata of exponential size $\sim\text{EXPTIME}$
Reasoning is \textit{EXPTIME}-complete

\textbf{Pre-completion for consistency:}

\begin{itemize}
  \item Happy is satisfiable to degree $p$ w.r.t. $O$ iff $O \cup \{\langle a: \text{Happy} \geq p \rangle\}$ is consistent
  \item Successful is subsumed by Happy to degree $p$ w.r.t. $O$ iff $O \cup \{\langle a: \text{Successful} \rightarrow \text{Happy} < p \rangle\}$ is inconsistent
\end{itemize}
Reasoning is \textsc{EXPTIME}-complete

Pre-completion for consistency:
Reasoning is EXPTIME-complete

Pre-completion for consistency:
Reasoning is EXP\textsc{TIME}-complete

Pre-completion for consistency:

Happy is satisfiable to degree $p$ w.r.t. $\mathcal{O}$

iff $\mathcal{O} \cup \{ \langle a : \text{Happy} \geq p \rangle \}$ is consistent

Successful is subsumed by Happy to degree $p$ w.r.t $\mathcal{O}$

iff $\mathcal{O} \cup \{ \langle a : \text{Successful} \rightarrow \text{Happy} < p \rangle \}$ is inconsistent
Conclusions

Summary:

- G-\textit{IALC} not as easy as thought, but still decidable
- General model semantics \cite{borgwardt2014general}
- Undecidable for all other t-norms \cite{borgwardt2012undecidable}

Future Work:

- \textit{PS \textit{PACE}} bounds for acyclic TBoxes \cite{borgwardt2013ps}
- Transitive and inverse roles, role hierarchy, nominals \cite{borgwardt2014transitive}
- Tableaux algorithm

Thank you
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- G\ödel\ extensions of small DLs easier
  (Borgwardt, Leyva Galano, and Peñaloza 2014; Mailis et al. 2012)

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