TECHNISCHE UNIVERSITÄT DRESDEN

CENTER FOR ADVANCING ELECTRONICS DRESDEN

DRESDEN concept

Institute of Theoretical Computer Science Chair of Automata Theory

DECIDABLE GÖDEL DESCRIPTION LOGICS WITHOUT THE FINITELY-VALUED MODEL PROPERTY

Stefan Borgwardt Felix Distel Rafael Peñaloza

Wien, July 23rd, 2014

Motivation

- vagueness often modeled ad-hoc, e.g. $\exists$severity.High in SNOMED CT

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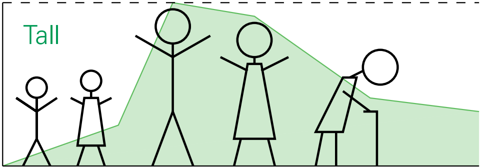

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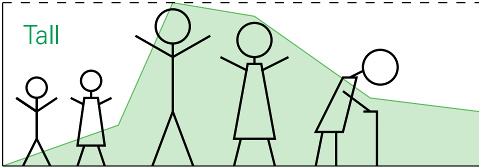

- mathematical fuzzy logic (Hájek 2001)
- combination of vague predicates via suitable functions

Happy $\sqcap$ Successful

Mathematical Fuzzy Logic

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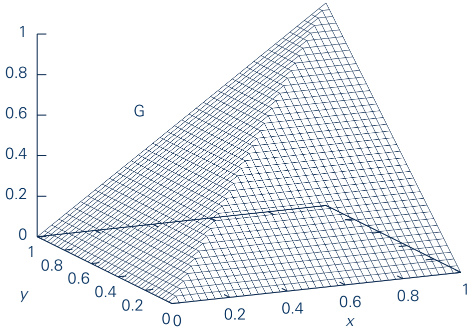

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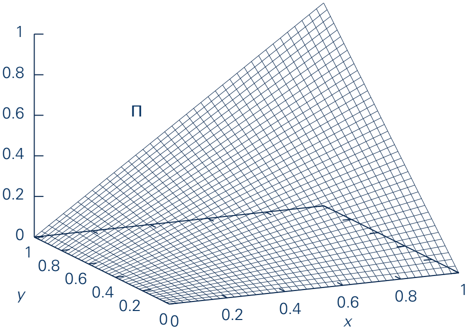

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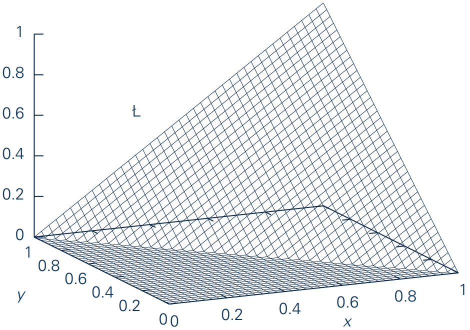
- Łukasiewicz (Ł): $\max(0, x + y - 1)$



The Fuzzy DL G-$\mathfrak{I}\mathcal{ALC}$

Fuzzy interpretations $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$:

- concept names: $\mathsf{Happy}^\mathcal{I} \colon \Delta^\mathcal{I} \to [0, 1]$
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Witnessed interpretations: (Hájek 2005)

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Axioms: order assertions

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- What is the best satisfiability/subsumption degree?

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FVMP $\equiv$ decidability ?

G-$\mathfrak{I}\mathcal{ALC}$ does not have the FVMP, but consistency is decidable.

The Finitely-Valued Model Property in G-$\mathcal{AL}$

$$x \Rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ y & \text{otherwise} \end{cases}$$

$\langle \text{felix} : \text{Happy} = 0.8 \rangle$ $\langle \forall \text{hasFriend}.\text{Happy} \sqsubseteq \text{Happy} \rangle$ $\langle \exists \text{hasFriend}.\top \sqsubseteq \text{Happy} \rangle$

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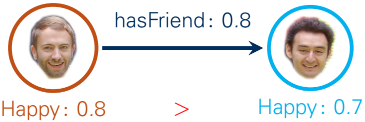

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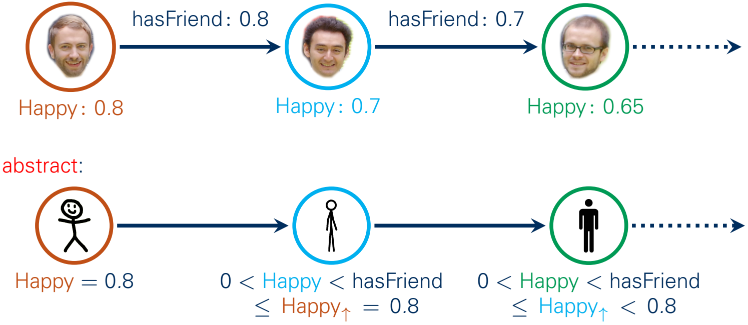

Only the Order Matters

abstract:

Happy $= 0.8$

$0 <$ Happy $<$ hasFriend $\leq \text{Happy}_{\uparrow} = 0.8$

$0 <$ Happy $<$ hasFriend $\leq \text{Happy}_{\uparrow} < 0.8$

Hintikka trees consisting of Hintikka orderings:

$$0 < \forall\text{hasFriend.Happy} < \text{Happy} \equiv \exists\text{hasFriend.}\top \equiv (\forall\text{hasFriend.Happy})_{\uparrow}$$
$$< \text{hasFriend} \equiv A_{\uparrow} \equiv (\exists\text{hasFriend.}\top)_{\uparrow} < 0.2 < 0.5 < 0.8 < 1 \equiv \top \equiv \top_{\uparrow}$$

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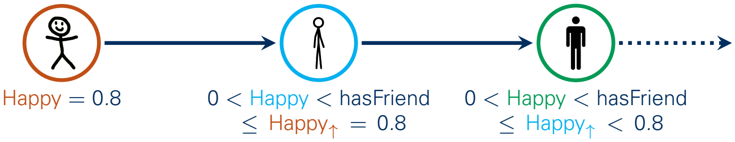

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looping tree automata of exponential size $\rightsquigarrow$ ExpTime

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Pre-completion for consistency:

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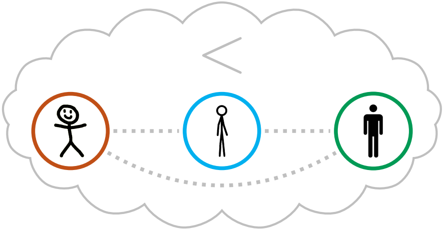

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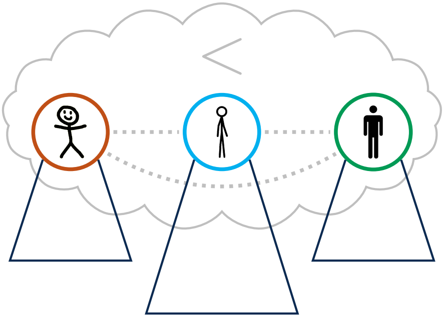

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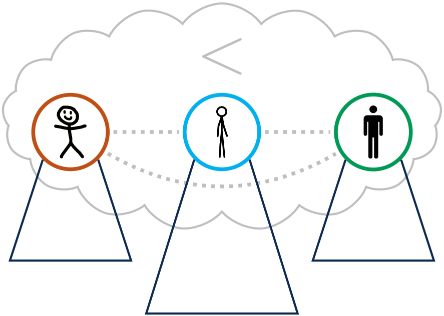

Happy is satisfiable to degree p w.r.t. $\mathcal{O}$

iff $\mathcal{O} \cup \{\langle a : \text{Happy} \geq p \rangle\}$ is consistent

Successful is subsumed by Happy to degree p w.r.t $\mathcal{O}$

iff $\mathcal{O} \cup \{\langle a : \text{Successful} \rightarrow \text{Happy} < p \rangle\}$ is inconsistent

Conclusions

Summary:

- G-$\mathfrak{I}\mathcal{ALC}$ not as easy as thought, but still decidable
- general model semantics (Borgwardt, Distel, and Peñaloza 2014)
- undecidable for all other t-norms (Borgwardt and Peñaloza 2012)

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- Baader, Franz and Rafael Peñaloza (2011). "On the Undecidability of Fuzzy Description Logics with GCIs and Product T-norm". In: *Proc. FroCoS'11*. Volume 6989. LNCS, pages 55–70.
- Bobillo, Fernando, Félix Bou, and Umberto Straccia (2011). "On the Failure of the Finite Model Property in some Fuzzy Description Logics". In: *Fuzzy Set. Syst.* 172.1, pages 1–12.
- Bobillo, Fernando and Umberto Straccia (2013). "Finite Fuzzy Description Logics and Crisp Representations". In: *Uncertainty Reasoning for the Semantic Web II*. Volume 7123. LNCS, pages 102–121.
- Borgwardt, Stefan (2014). "Fuzzy DLs over Finite Lattices with Nominals". In: *Proc. DL'14*. Volume 1193. CEUR-WS, pages 58–70.
- Borgwardt, Stefan, Felix Distel, and Rafael Peñaloza (2012). "How Fuzzy is my Fuzzy Description Logic?" In: *Proc. IJCAR'12*. Volume 7364. LNAI, pages 82–96.
- — (2014). "Gödel Description Logics with General Models". In: *Proc. DL'14*. Volume 1193. CEUR-WS. Poster paper., pages 391–403.
- Borgwardt, Stefan, José A. Leyva Galano, and Rafael Peñaloza (2014). "Gödel $\mathcal{FL}_0$ with Greatest Fixed-Point Semantics". In: *Proc. DL'14*. Volume 1193. CEUR-WS, pages 71–82.
- Borgwardt, Stefan and Rafael Peñaloza (2012). "Undecidability of Fuzzy Description Logics". In: *Proc. KR'12*, pages 232–242.
- — (2013). "The Complexity of Lattice-Based Fuzzy Description Logics". In: *J. Data Semant.* 2.1, pages 1–19.
- Cerami, Marco and Umberto Straccia (2013). "On the (Un)decidability of Fuzzy Description Logics under Łukasiewicz t-norm". In: *Inform. Sci.* 227, pages 1–21.
- Hájek, Petr (2001). *Metamathematics of Fuzzy Logic (Trends in Logic)*.

References II

- — (2005). "Making Fuzzy Description Logic more General". In: *Fuzzy Set. Syst.* 154.1, pages 1–15.
- Mailis, Theofilos, Giorgos Stoilos, Nikolaos Simou, Giorgos B. Stamou, and Stefanos Kollias (2012). "Tractable Reasoning with Vague Knowledge using Fuzzy $\mathcal{EL}^{++}$". In: *J. Intell. Inf. Syst.* 39.2, pages 399–440.
- Zadeh, Lotfi A. (1965). "Fuzzy Sets". In: *Inform. Control* 8.3, pages 338–353.