TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

Institute of Theoretical Computer Science Chair of Automata Theory

FINDING FINITE HERBRAND MODELS

Stefan Borgwardt Barbara Morawska

Mérida, March 14, 2012

Introduction

- satisfiability problem for first-order formulae: $\forall x.P(f(x))$
- Herbrand model: $P(f(a)), P(f(f(a))), \ldots$

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- predicates are interpreted by finite sets
- recursively enumerable
- EXPTIME-complete for a subset of FOL
- automata-like model expresses the model-building process

Propagation Rules

Signature: Unary predicates, unary functions, one constant a, one variable x.

$$\begin{aligned} \top &\rightarrow P_1(a) \vee \cdots \vee P_n(a) \\ P_0(a) &\rightarrow P_1(a) \vee \cdots \vee P_n(a) \\ P_0(t_0) &\rightarrow P_1(t_1) \vee \cdots \vee P_n(t_n) \end{aligned}$$

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Two exceptions:

- increasing clauses: $P^f(x) \to P(f(x))$, $P^{fg}(x) \to P^f(g(x))$
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$\mathcal{C}$ …finite set of propagation rules
$\mathcal{D}(\mathcal{C})$ …set of predicates of the form P^f in $\mathcal{C}$

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$$P_Y(r(x)) \to P_Y(x),\ P_Z(x) \to P_Y(x)$$

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ExpTime-hard $\Leftarrow$ ExpTime-complete [Baader and Narendran, 2001]

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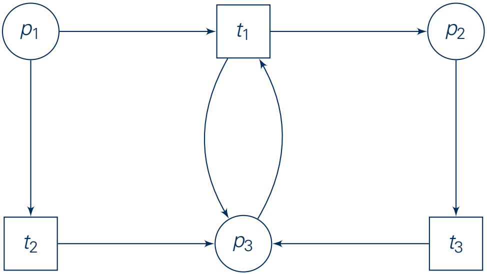
best-case exponential decision procedure

Propagation Nets

$\mathcal{N} = (P, T, \Sigma, E, l, \pi, \tau)$:

- places P, transitions T, input symbols Σ, arcs $E \subseteq (P \times T) \cup (T \times P)$,
- words from Σ^* ... tokens,



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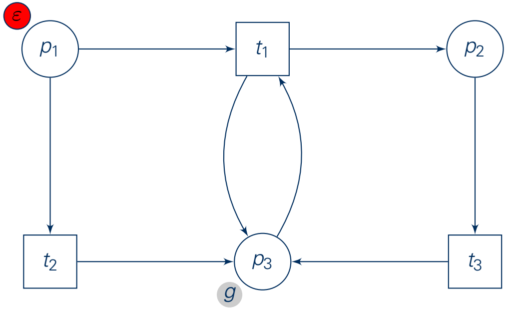

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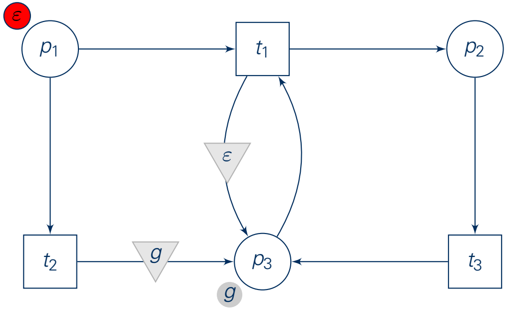

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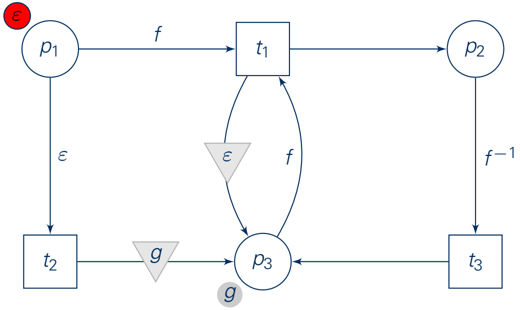

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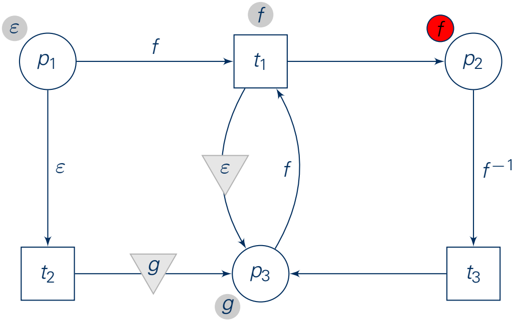

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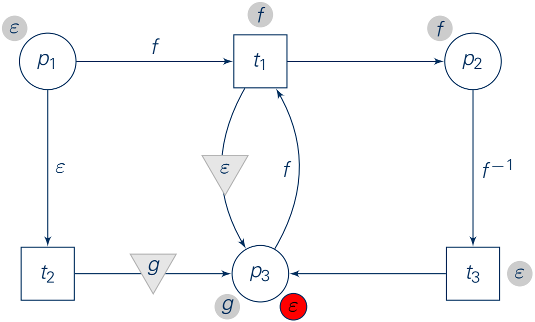

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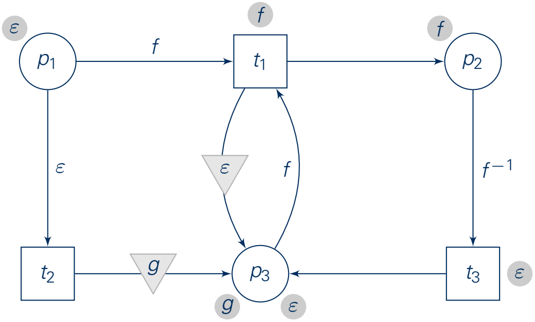

Two-way Alternating Tree Automata

Alternative point of view:

places	=	existential states
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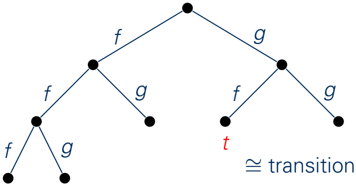

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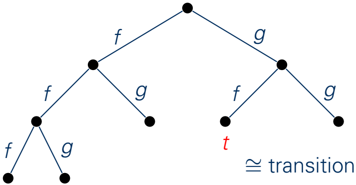

≅ transition *t* is marked by *fg*

Main difference:

- finite tree automata require runs to be finite
- in propagation nets cycles are allowed

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Tokens ε, f, gf

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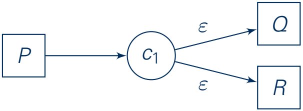

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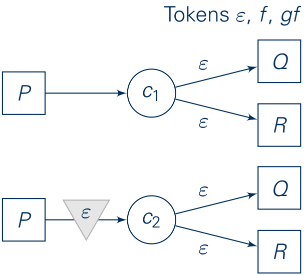

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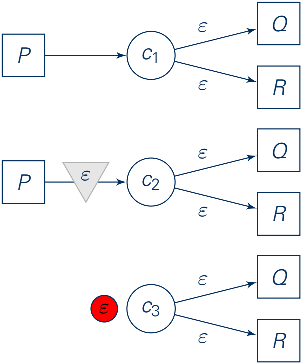

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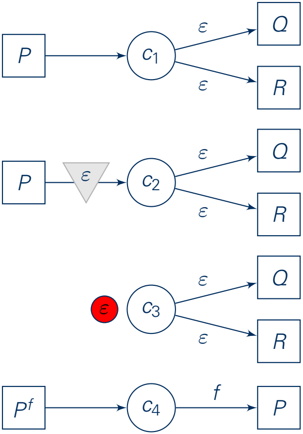

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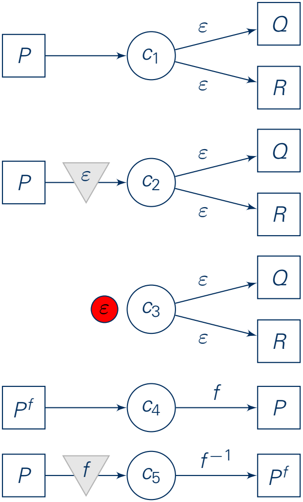
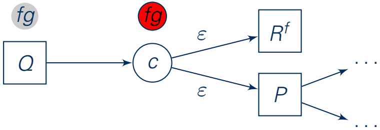
$c_5 : P(f(x)) \to P^f(x)$



Firing Sequences

Firing: $(Q(x) \to R^f(x) \lor P(x), fg, P)$, abbreviated $Q(fg) \to P(fg)$



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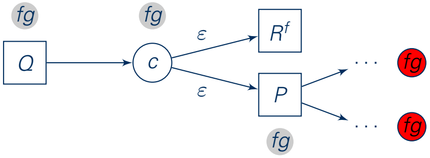

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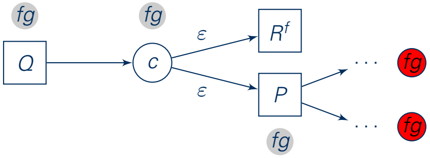

Firing sequence:

$$R(fg) \longrightarrow Q(fg) \longrightarrow P(fg)$$

$$\nearrow \qquad \searrow \qquad \searrow$$

$$Q(g) \longrightarrow R^f(g) \qquad Q^f(g) \qquad P^f(g)$$

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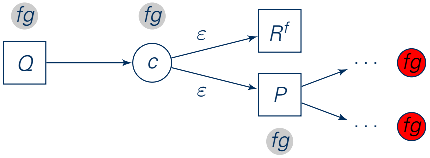

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Lemma

$\mathcal{C}$ has a finite Herbrand model iff $\mathcal{N}_{\mathcal{C}}$ has a terminating firing sequence.

The Decision Procedure

$$\top \to Q^g(a) \vee R^g(a) \qquad R(x) \to Q(x) \qquad P^g(x) \to \bot$$
$$Q(x) \to R^f(x) \vee P(x) \qquad Q^g(x) \to P(x) \vee R^g(x)$$

$$\mathcal{D}(\mathcal{C}) = \{P^f, P^g, Q^f, Q^g, R^f, R^g\}$$

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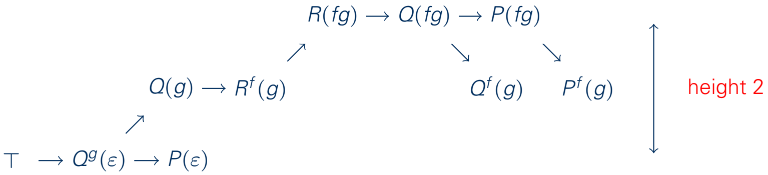

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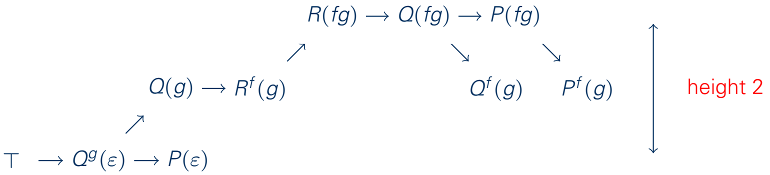

Idea:
Use firing sequences of small height to build firing sequences of larger height.

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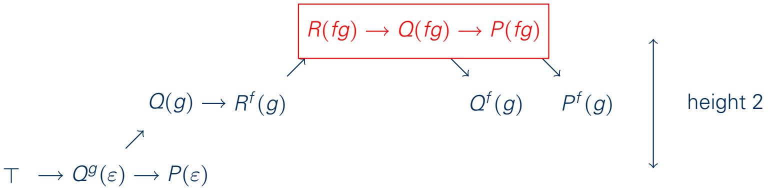

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Shortcuts: $(R, \{P, Q, R\})$

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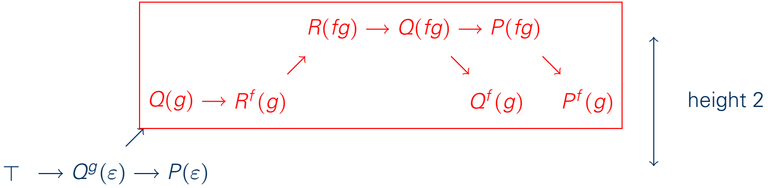

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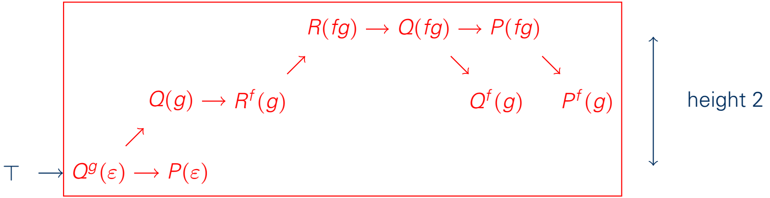

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Main Algorithm

Input: a finite set $\mathcal{C}$ of propagation rules
Output: `true` iff $\mathcal{N}_{\mathcal{C}}$ terminates

```
R_0 ← ∅, i ← 0
repeat
    if isTerminating(C, R_i) then return true
    R_{i+1} ← nextShortcuts(C, R_i)
    i ← i + 1
until R_i = R_{i-1}
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$\mathtt{nextShortcuts}(\mathcal{C}, \mathcal{R}_i)$: Given all shortcuts $\mathcal{R}_i$ for height at most $i - 1$, compute the shortcuts for height at most i.

Complexity

Exponential Time:

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Mixed Nets:

- exponential in the number of nondeterministic clauses
  (every nondeterministic clause adds a constant factor)
- polynomial in the number of deterministic clauses

Conclusions

Summary:

- worst-case exponential algorithm for ...
  - finding finite Herbrand models for sets of propagation rules
  - solving linear language inclusions
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Future Work:

- actually compute finite Herbrand models
- consider arbitrary clauses with unary predicates and function symbols
- other applications?

Thank You

Franz Baader and Paliath Narendran. Unification of concept terms in description logics. *J. Symb. Comput.*, 31(3):277–305, 2001.

Stefan Borgwardt and Barbara Morawska. Finding finite Herbrand models. LTCS-Report 11-04, Chair for Automata Theory, TU Dresden, Germany, 2011. See http://lat.inf.tu-dresden.de/research/reports.html.

$\texttt{isTerminating}(\mathcal{C}, \mathcal{R})$

We assume that there is only one positive clause $\top \rightarrow A(a)$. A predicate P is good iff there is a terminating firing sequence starting with $P(\varepsilon)$.

Input: a finite set $\mathcal{C}$ of propagation rules and a set $\mathcal{R}$ of shortcuts
Output: `true` iff A is good w.r.t. $\mathcal{R}$

$\mathcal{B}_0 \leftarrow \emptyset$, $k \leftarrow 0$
repeat

$$\begin{aligned}\mathcal{B}_{k+1} \leftarrow\ & \mathcal{B}_k \\ & \cup \{P \in \mathcal{P} \mid \exists P(x) \rightarrow P_1(x) \vee \cdots \vee P_n(x) \in \mathcal{C} : \{P_1, \ldots, P_n\} \subseteq \mathcal{B}_k\} \\ & \cup \{P \in \mathcal{P} \mid \exists P(a) \rightarrow P_1(a) \vee \cdots \vee P_n(a) \in \mathcal{C} : \{P_1, \ldots, P_n\} \subseteq \mathcal{B}_k\} \\ & \cup \{P^f \in \mathcal{P} \mid (P, f) \in \mathcal{D}(\mathcal{C}),\ \forall (P, \mathcal{X}) \in \mathcal{R}\ \exists Q \in \mathcal{X} \cap \mathcal{D}^f(\mathcal{C}) : Q^f \in \mathcal{B}_k\}\end{aligned}$$

$k \leftarrow k + 1$
until $\mathcal{B}_k = \mathcal{B}_{k-1}$
return $A \notin \mathcal{B}_k$

Possibilities

$$\top \rightarrow Q^g(a) \vee R^g(a) \qquad R(x) \rightarrow Q(x) \qquad P^g(x) \rightarrow \bot$$

$$Q(x) \rightarrow R^f(x) \vee P(x) \qquad Q^g(x) \rightarrow P(x) \vee R^g(x)$$

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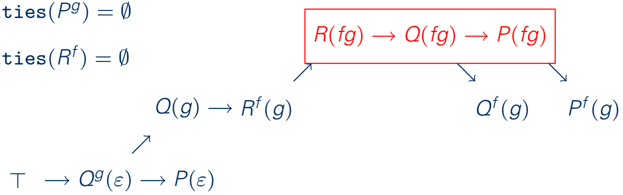

If the shortcut $(R, \{P, Q, R\})$ is known: $\texttt{possibilities}(R^f) = \{\{P^f, Q^f, R^f\}\}$

possibilities($\mathcal{C}, \mathcal{R}, P$)

Input: a finite set $\mathcal{C}$ of propagation rules, a set $\mathcal{R}$ of shortcuts, and a predicate P
Output: the set of possibilities for P w.r.t. $\mathcal{C}$ and $\mathcal{R}$

if $P = Q^f$ with $(Q, f) \in \mathcal{D}(\mathcal{C})$ **then**
  $\mathcal{L} \leftarrow \{\{Q_1^f, \ldots, Q_n^f\} \mid (Q, \mathcal{X}) \in \mathcal{R},\ \{Q_1, \ldots, Q_n\} = \mathcal{X} \cap \mathcal{D}^f(\mathcal{C})\}$
else $\mathcal{L} \leftarrow \{\emptyset\}$
for all $P(x) \rightarrow P_1(x) \vee \cdots \vee P_n(x) \in \mathcal{C}$ **do**
  $\mathcal{L} \leftarrow \{\mathcal{Y} \cup \{P_l\} \mid \mathcal{Y} \in \mathcal{L},\ l \in \{1, \ldots, n\}\}$
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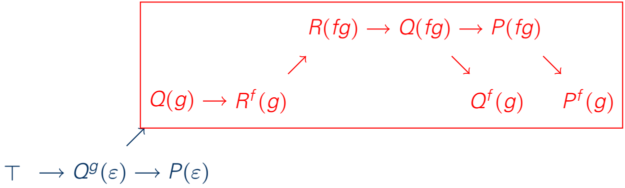
Shortcut: $(Q, \{Q, P^f, R^f, Q^f\})$

Computing Shortcuts



$\texttt{nextShortcuts}(\mathcal{C}, \mathcal{R})$:

Input: a finite set $\mathcal{C}$ of propagation rules,
a set $\mathcal{R}$ of shortcuts
Output: a set of shortcuts for the next height

$\mathcal{T} \leftarrow \{(P, \{P\}, \emptyset) \mid \exists P^f \in \mathcal{D}(\mathcal{C})\}$
while $\exists (P, R_P, V_P) \in \mathcal{T}$ with $R_P \setminus V_P \neq \emptyset$ **do**
  $\mathcal{T} \leftarrow \mathcal{T} \setminus \{(P, R_P, V_P)\}$
  choose Q from $R_P \setminus V_P$
  for all $\mathcal{Y} \in \texttt{possibilities}(\mathcal{C}, \mathcal{R}, Q)$ **do**
    $\mathcal{T} \leftarrow \mathcal{T} \cup \{(P, R_P \cup \mathcal{Y}, V_P \cup \{Q\})\}$
return $\{(P, R_P) \mid (P, R_P, R_P) \in \mathcal{T}\}$

$(Q, \{Q\}, \emptyset)$
$(Q, \{Q, R^f\}, \{Q\})$
$(Q, \{Q, R^f, Q^f, P^f\}, \{Q, R^f\})$
$(Q, \{Q, R^f, Q^f, P^f\}, \{Q, R^f, Q^f\})$
$(Q, \{Q, R^f, Q^f, P^f\}, \{Q, R^f, Q^f, P^f\})$
Shortcut: $(Q, \{Q, P^f, R^f, Q^f\})$