TECHNISCHE UNIVERSITÄT DRESDEN

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Fuzzy Description Logics and Probabilistic Databases

Dresden, 26th June 2018

Fuzzy Description Logics

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- Represent vague concepts and roles

CheapHotel, isNear, Fuzzy

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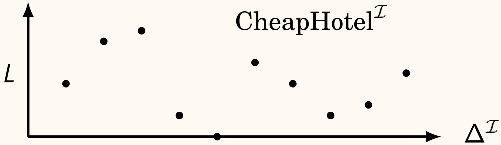
- Fuzzy concepts interpreted as fuzzy sets $\Delta^{\mathcal{I}} \to L$



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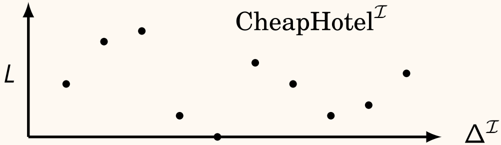

- Syntax remains the same

$\mathcal{ALC}$: $\top$, $\bot$, $\sqcap$, $\sqcup$, $\neg$, $(\to)$, $\exists r$, $\forall r$

Fuzzy Semantics over $[0, 1]$

- Truth-functionality: truth degree of concepts computed recursively

$$(\mathrm{Cheap} \sqcap \mathrm{Hotel})^{\mathcal{I}}(d) = \mathrm{Cheap}^{\mathcal{I}}(d) \otimes \mathrm{Hotel}^{\mathcal{I}}(d)$$

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- Concept interpretations are lifted according to fuzzy first-order logic

$$d \in (\forall r.C)^{\mathcal{I}} \quad \text{iff} \quad \forall e \in \Delta^{\mathcal{I}}.\, (d,e) \in r^{\mathcal{I}} \to e \in C^{\mathcal{I}}$$

$$(\forall r.C)^{\mathcal{I}}(d) \quad = \quad \inf_{e \in \Delta^{\mathcal{I}}} r^{\mathcal{I}}(d,e) \Rightarrow C^{\mathcal{I}}(e)$$

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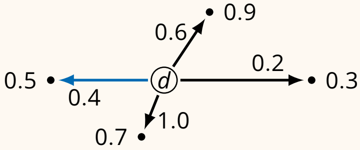
$(\forall \mathrm{isNear}.\mathrm{CheapHotel})^{\mathcal{I}}(d) = 0.6$:



Other Fuzzy Semantics over [0, 1]

- Conjunctions interpreted by t-norms, implications by their residua

	Conjunction $x \otimes y$	Implication $x \Rightarrow y$	Negation $\ominus x := x \Rightarrow 0$
Zadeh	$\min\{x, y\}$	$\max\{1 - x, y\}$	$1 - x$
Gödel	$\min\{x, y\}$	$\begin{cases} 1 & \text{if } x \leq y \\ y & \text{if } x > y \end{cases}$	$\begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x > 0 \end{cases}$
Product	$x \cdot y$	$\begin{cases} 1 & \text{if } x \leq y \\ y/x & \text{if } x > y \end{cases}$	$\begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x > 0 \end{cases}$
Łukasiewicz	$\max\{x + y - 1, 0\}$	$\min\{1 - x + y, 1\}$	$1 - x$

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- Fuzzy axioms encode fuzzy knowledge

Concept inclusions:	$\langle \mathrm{PopularPlace} \sqsubseteq \forall \mathrm{isNear}.\neg \mathrm{CheapHotel} \geq 0.7 \rangle$
Inequality assertions:	$\langle \mathrm{CheapHotel}(\mathrm{xyzHotel}) \geq 0.7 \rangle$
Equality assertions:	$\langle \mathrm{CheapHotel}(\mathrm{xyzHotel}) = 0.7 \rangle$

Complexity of Fuzzy $\mathcal{ALC}$

Semantics	Inequality assertions	Equality assertions
Zadeh	ExpTime	ExpTime
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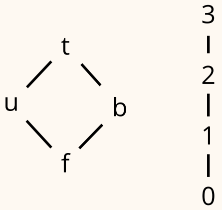
- (non-trivially) reducible to classical $\mathcal{ALC}$ (Bobillo, Delgado, Gómez-Romero, and Straccia 2012; Borgwardt and Peñaloza 2017)
- undecidable even for $\mathcal{EL}$ (Borgwardt, Cerami, and Peñaloza 2017)

Finitely-Valued Fuzzy $\mathcal{ALC}$

- Finite lattices L allow incomparable degrees of truth



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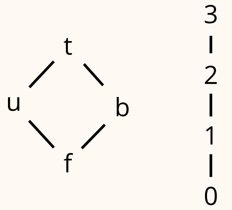

- Can be reduced to classical DLs via cuts

$\mathrm{isNear}_{\geq 2}$ $\quad$ $\mathrm{InterestingTalk}_{\geq \mathrm{u}}$

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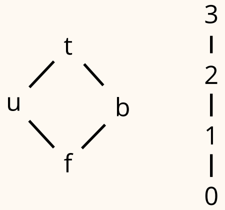

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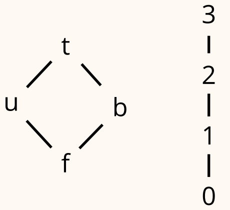

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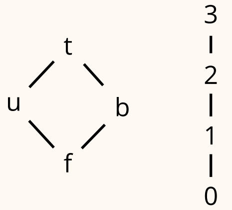

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  - direct tableaux algorithm (Straccia 2006; Borgwardt and Peñaloza 2014)


Google’s Knowledge Vault
NELL
DeepDive


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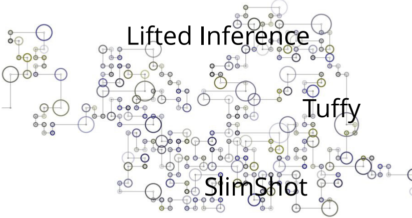
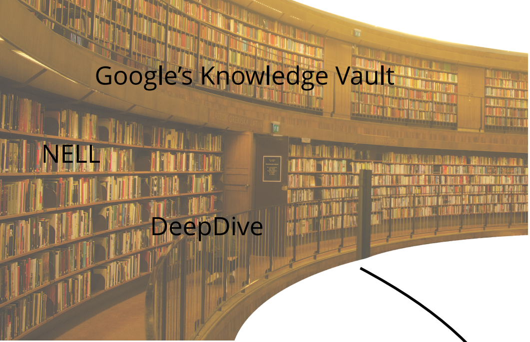
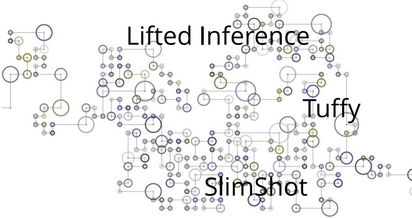

Lifted Inference
Tuffy
SlimShot





Probabilistic Databases (PDBs)

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Banks	0.8
Hamilton	0.7

AuthorOf		
Banks	Excession	0.9
Banks	Inversions	0.8
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Probabilistic query evaluation

$$\mathrm{Q}: \quad \exists x.\, \mathrm{AuthorOf}(\mathrm{Banks}, x)$$

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$$\mathrm{P}(Q) = \sum_{\mathcal{D} \models Q} \mathrm{P}(\mathcal{D})$$

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$$\mathrm{P}(\mathrm{Writer}(\mathrm{Stephenson})) = 0$$

Closed-world

Ontology-Mediated Probabilistic Queries

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$$\exists \text{AuthorOf}.\text{Novel} \sqsubseteq \text{Writer}$$

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Ontology-Mediated Queries (OMQs)

(conjunctive query Q , ontology $\mathcal{O}$)

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$$\mathrm{P}(\mathsf{Q}, \mathcal{O}) = \sum_{\mathcal{D} \models (\mathsf{Q}, \mathcal{O})} \mathrm{P}(\mathcal{D})$$

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Not tuple-independent

Rewritability

UCQ-rewritability of $(\mathsf{Q}, \mathcal{O})$: $\mathcal{D} \models (\mathsf{Q}, \mathcal{O})$ iff $\mathcal{D} \models \mathsf{Q}_{\mathcal{O}}$

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$\rightsquigarrow$ Probability of $(\mathsf{Q},\mathcal{O})$: $\mathrm{P}(\mathsf{Q},\mathcal{O}) = \mathrm{P}(\mathsf{Q}_{\mathcal{O}})$

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Dichotomy

For UCQ-rewritable $(\mathsf{Q}, \mathcal{O})$,
deciding $\mathrm{P}(\mathsf{Q}, \mathcal{O}) > p$ is either in P ("safe") or PP-complete ("unsafe").

(Dalvi and Suciu 2012)
(Jung and Lutz 2012; Borgwardt, Ceylan, and Lukasiewicz 2017)

Safe and Unsafe Queries

$$\begin{aligned} \mathcal{O}\text{:} \qquad \exists \text{AuthorOf}.\text{Novel} &\sqsubseteq \text{Writer} \\ \exists \text{CharacterIn}.\text{Novel} &\sqsubseteq \text{LiteraryCharacter} \end{aligned}$$

Safe → Unsafe

Q: $\exists x.\ \text{Writer}(x) \wedge \text{LiteraryCharacter}(x)$

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Safe → Unsafe

Q: $\exists x.\, \text{Writer}(x) \land \text{LiteraryCharacter}(x)$

$\rightsquigarrow$

$\text{Q}_{\mathcal{O}}$: $\exists x, y, z.\, \text{AuthorOf}\,(x, y) \land \text{Novel}(y) \land \text{CharacterIn}(x, z) \land \text{Novel}(z) \ldots$

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$Q_{\mathcal{O}}$: $\exists x, y, z.\, \text{AuthorOf}(x, y) \land \text{Novel}(y) \land \text{CharacterIn}(x, z) \land \text{Novel}(z) \ldots$

Unsafe → Safe

Q: $\exists x, y.\, \text{Writer}(x) \land \text{AuthorOf}(x, y) \land \text{Novel}(y)$

Safe and Unsafe Queries

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$$\rightsquigarrow$$

$\mathrm{Q}_{\mathcal{O}}$: $\exists x, y, z.\, \mathrm{AuthorOf}(x, y) \land \mathrm{Novel}(y) \land \mathrm{CharacterIn}(x, z) \land \mathrm{Novel}(z) \ldots$

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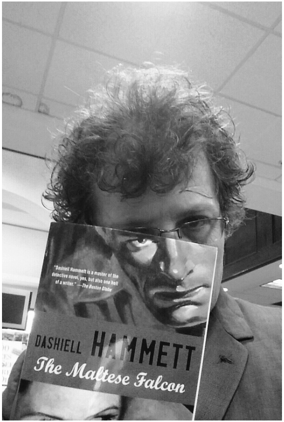
Inconsistency

$$\mathsf{Writer} \sqcap \mathsf{Novel} \sqsubseteq \bot$$



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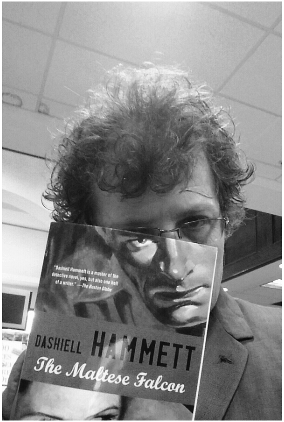

If $\mathcal{D} \models (\bot, \mathcal{O})$, then $\mathcal{D} \models (\mathsf{Q}, \mathcal{O})$ trivially holds.

Dealing With Inconsistency

Normalization

$$\mathrm{P_n}(\mathsf{Q}, \mathcal{O}) = \frac{\mathrm{P}(\mathsf{Q}, \mathcal{O}) - \mathrm{P}(\bot, \mathcal{O})}{1 - \mathrm{P}(\bot, \mathcal{O})}$$

Dealing With Inconsistency

Normalization

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After rewriting: $$\mathrm{P}_n(Q, \mathcal{O}) = \frac{\mathrm{P}(Q_{\mathcal{O}}) - \mathrm{P}(Q_{\mathcal{O}} \wedge \bot_{\mathcal{O}})}{1 - \mathrm{P}(\bot_{\mathcal{O}})}$$

Dealing With Inconsistency

Normalization

$$\mathrm{P}_{\mathsf{n}}(\mathsf{Q},\mathcal{O}) = \frac{\mathrm{P}(\mathsf{Q},\mathcal{O}) - \mathrm{P}(\bot,\mathcal{O})}{1 - \mathrm{P}(\bot,\mathcal{O})}$$

After rewriting: $$\mathrm{P}_{\mathsf{n}}(\mathsf{Q},\mathcal{O}) = \frac{\mathrm{P}(\mathsf{Q}_{\mathcal{O}}) - \mathrm{P}(\mathsf{Q}_{\mathcal{O}} \wedge \bot_{\mathcal{O}})}{1 - \mathrm{P}(\bot_{\mathcal{O}})}$$

Tuple probabilities are distorted, even if they are not affected by the cause of the inconsistency (or other axioms)

Dealing With Inconsistency (II)

Alternative:

Maximum Entropy Approach

$$\textit{Maximize: } H(\mathrm{P_m}) = -\sum_{\mathcal{D}} \mathrm{P_m}(\mathcal{D}) \log \mathrm{P_m}(\mathcal{D})$$

$$\textit{Subject to: } \sum_{\mathcal{D}} \mathrm{P_m}(\mathcal{D}) = 1$$

$$\mathrm{P_m}(\mathcal{D}) = 0 \text{ if } \mathcal{D} \models (\bot, \mathcal{O})$$

$$\mathrm{P_m}(t) = p \text{ for all } \langle t : p \rangle \in \mathcal{P}$$

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- May be infeasible $\rightsquigarrow$ use slack variables
- Challenging to compute $\mathrm{P_m}$; area of active research
- Rewriting-based approaches possible

Summary

Ontologies modeling can benefit from finitely many fuzzy degrees.

Probabilistic databases can benefit from ontologies.

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Ontologies modeling can benefit from finitely many fuzzy degrees.

Probabilistic databases can benefit from ontologies.

Thank you!

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