



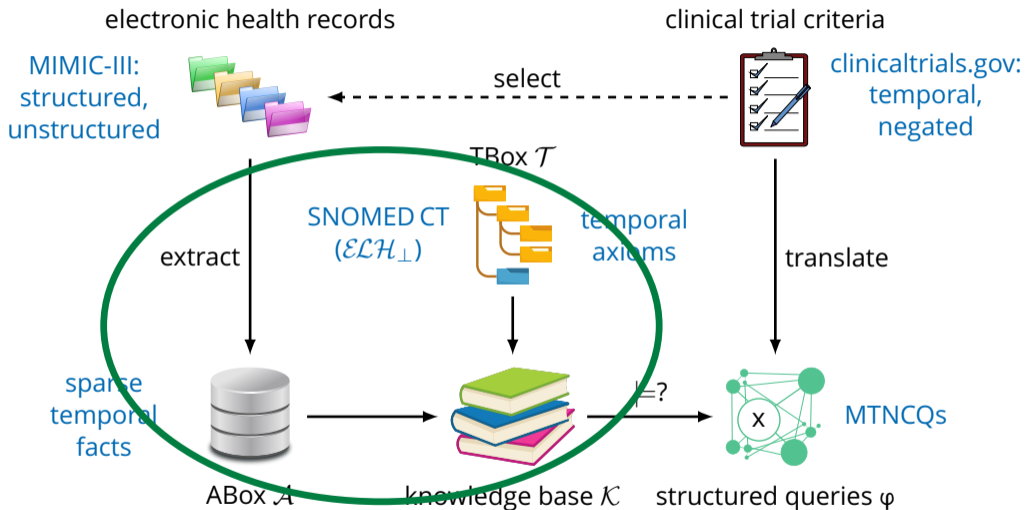
TECHNISCHE
UNIVERSITÄT
DRESDEN

Stefan Borgwardt, Walter Forkel, Alisa Kovtunova
Institute of Theoretical Computer Science, Chair of Automata Theory

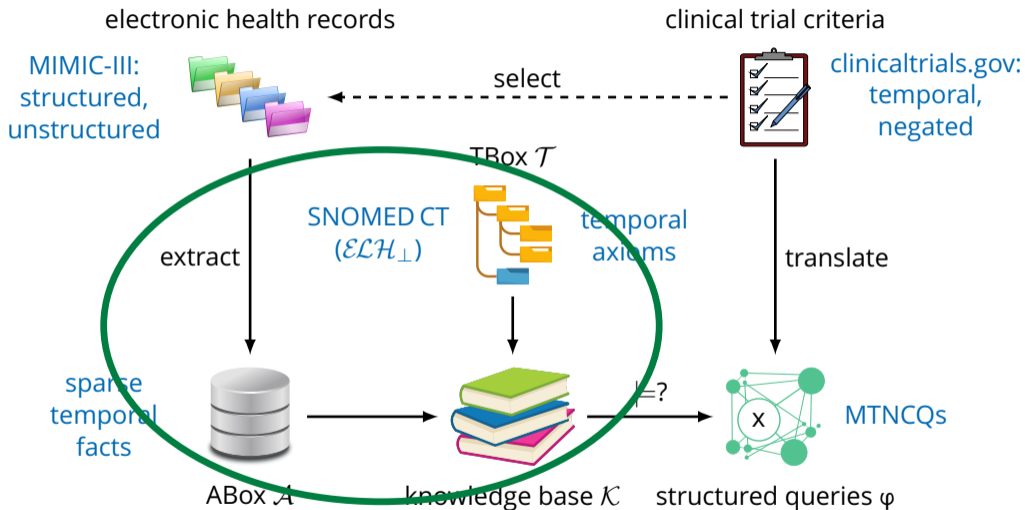
Finding New Diamonds: Temporal Minimal-World Query Answering over Sparse ABoxes

Bozen-Bolzano, 17th September 2019

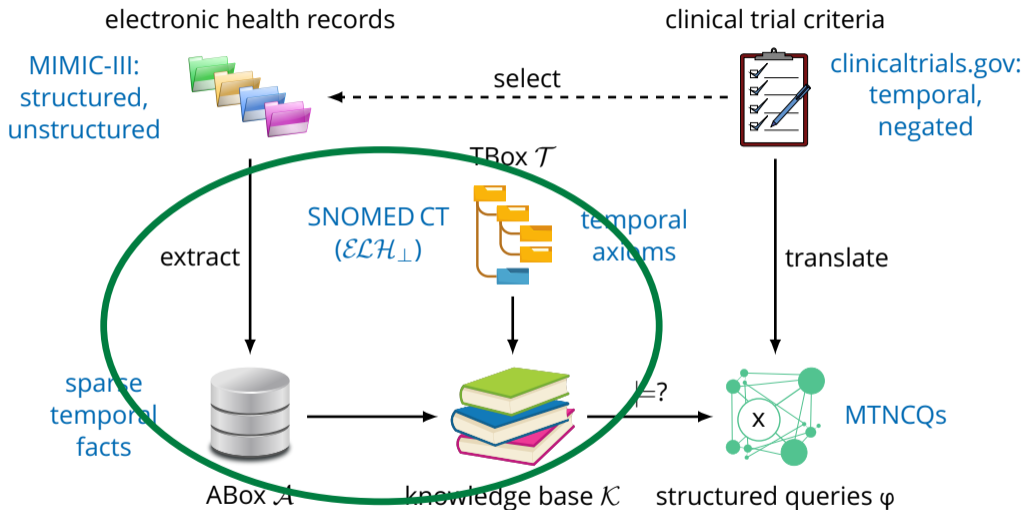
Motivation



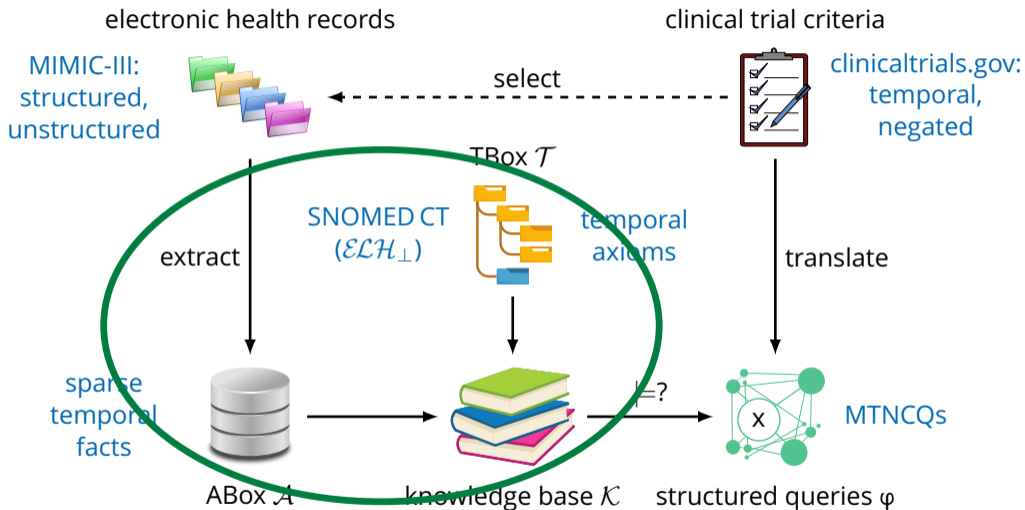
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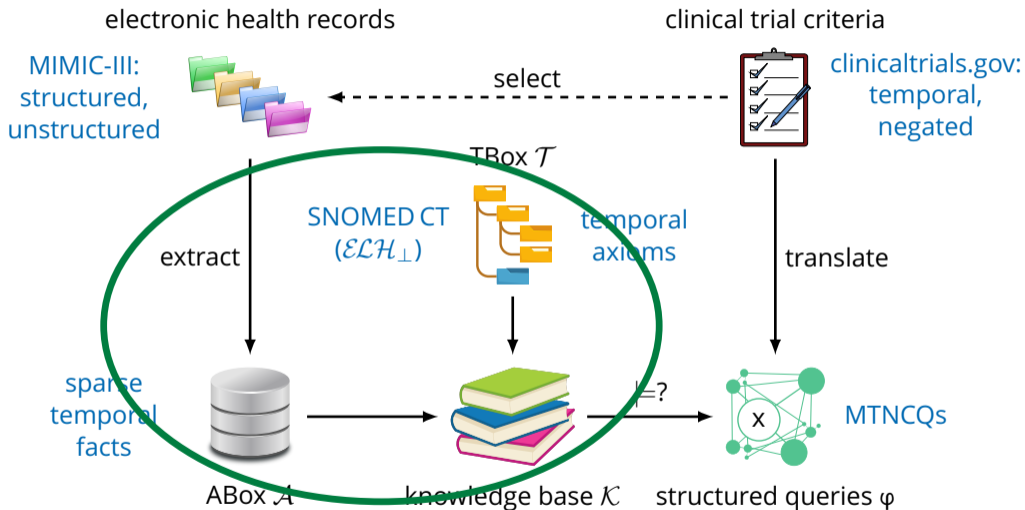
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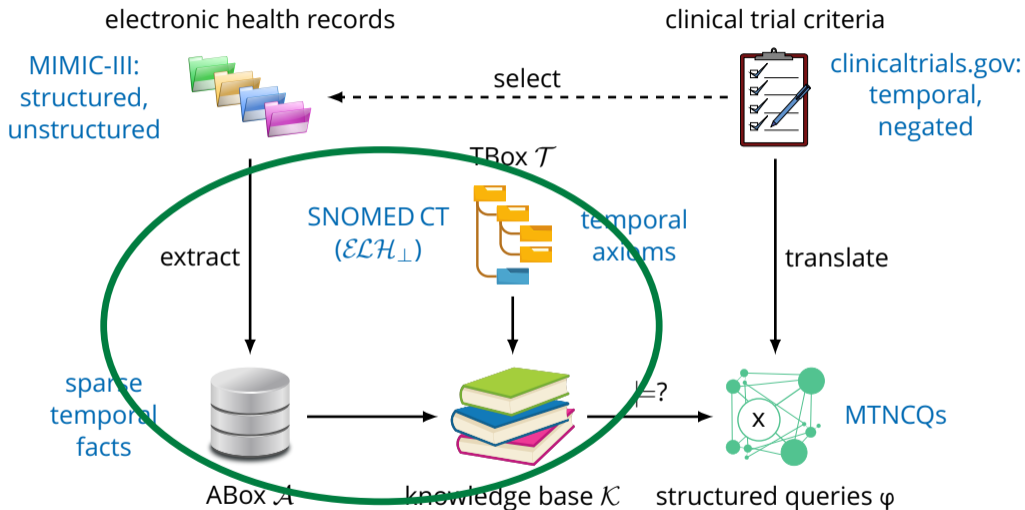
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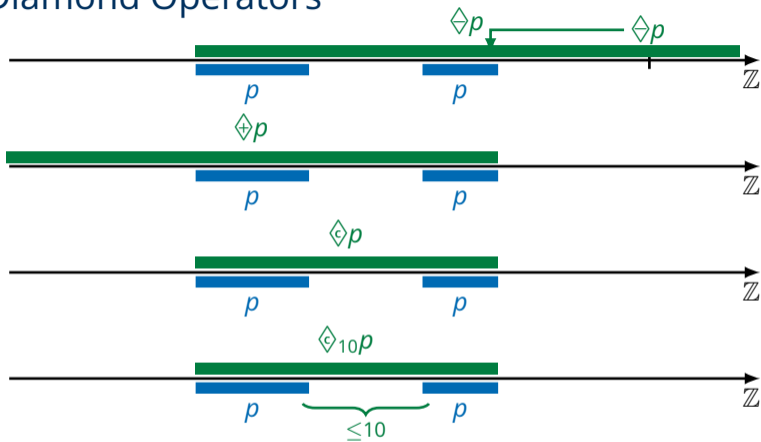
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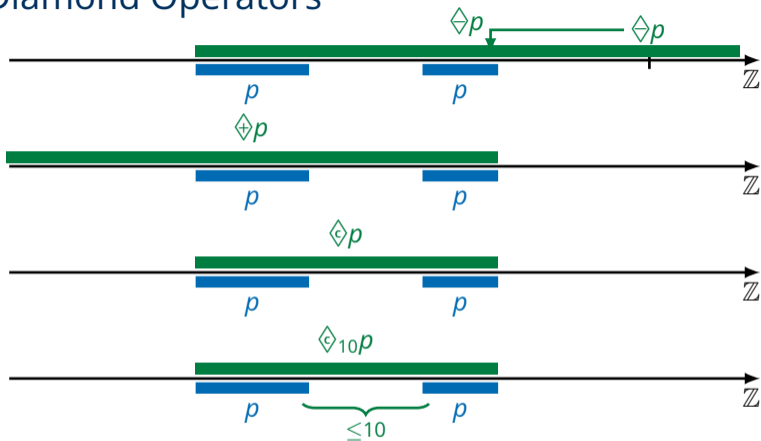
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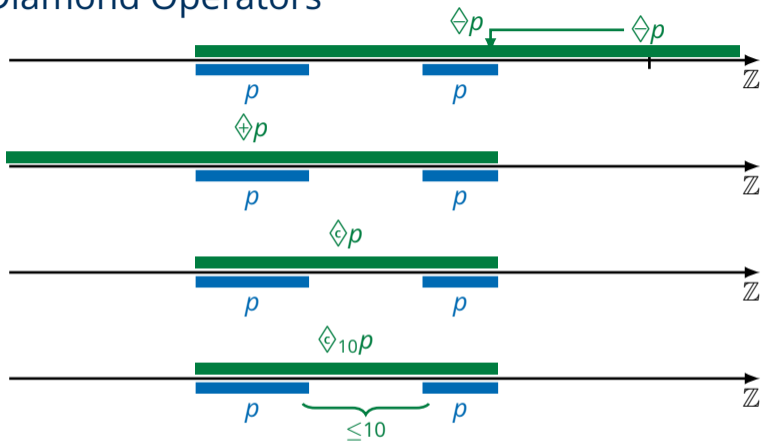
Metric Diamond Operators



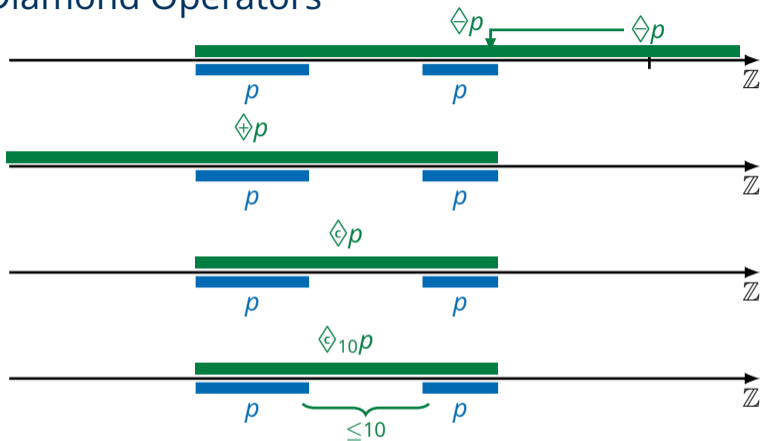
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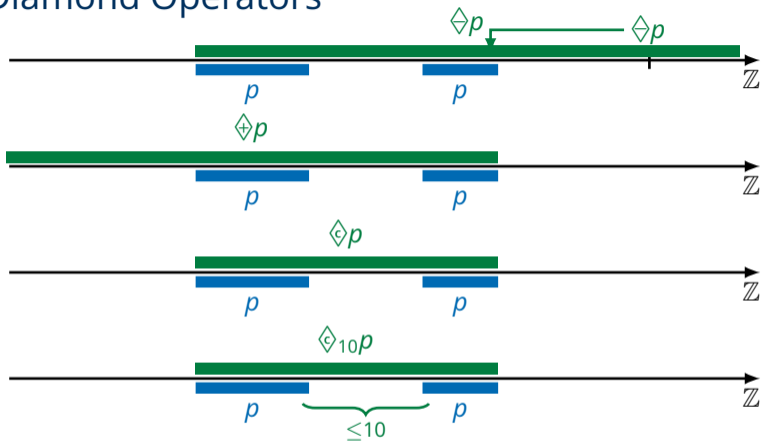
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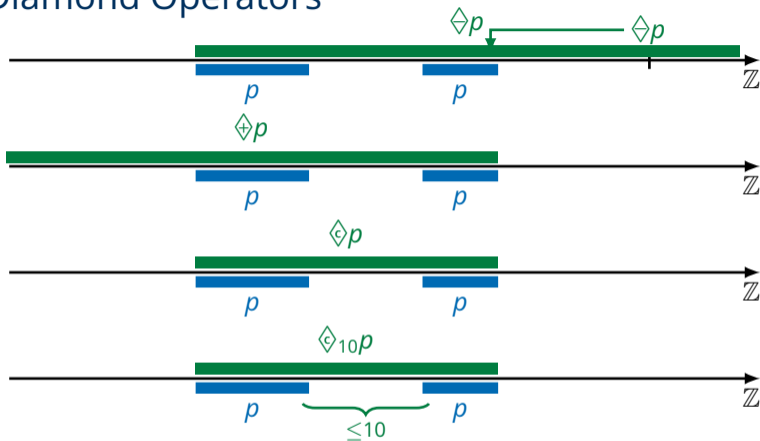
Metric Diamond Operators



Metric Diamond Operators



Metric Diamond Operators



- can be seen as closure operators $2^{\mathbb{Z}} \rightarrow 2^{\mathbb{Z}}$
- $\{\diamond_1, \diamond_2, \dots, \diamond, \diamond, \diamond, \diamond\}$ is closed under pointwise \cap, \cup, \circ

A Temporal Description Logic

based on (Gutiérrez-Basulto, Jung, and Kontchakov 2016)

$\diamond \exists \text{diagnosis.Diabetes} \sqsubseteq \exists \text{diagnosis.Diabetes}$

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$\mathcal{TECH}_{\perp}^{\diamond, \text{lhs}}$, a combination of \mathcal{ELH}_{\perp} with diamond operators:

$\diamond A \sqsubseteq B, A_1 \sqcap A_2 \sqsubseteq B, \diamond r \sqsubseteq s, \diamond A \sqsubseteq \exists r.B, \exists r.A \sqsubseteq B, A(a, i), r(a, b, i)$

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Translation into first-order logic:

$$\forall x, t, t'. A(x, t') \wedge (t' \leq t) \rightarrow \exists y. r(x, y, t) \wedge B(y, t)$$

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Theorem

Entailment in $\mathcal{TELH}_{\perp}^{\diamond, \text{lhs}}$ is P-complete.

(Some) Completion Rules

$$\frac{\diamond^* A_1 \sqsubseteq A_2 \quad \diamond A_2 \sqsubseteq A_3}{(\diamond \circ \diamond^*) A_1 \sqsubseteq A_3}$$

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(only for **representative time points i**)

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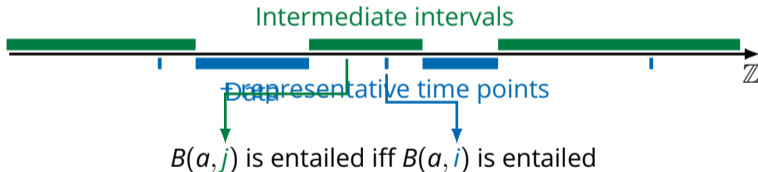
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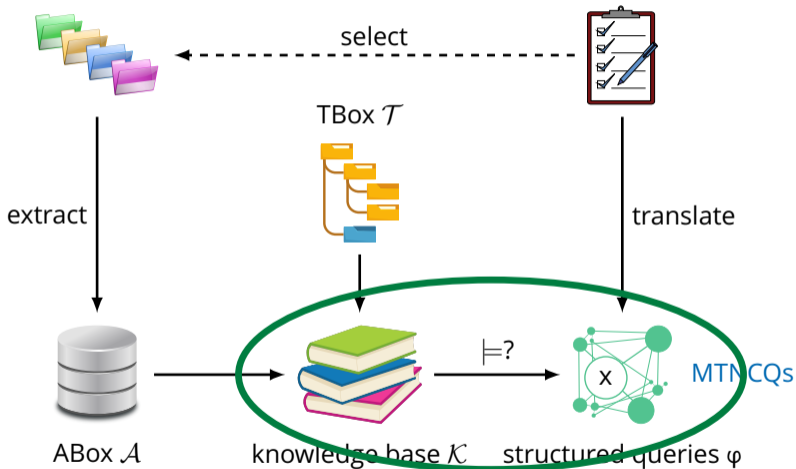
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Flashback

electronic health records

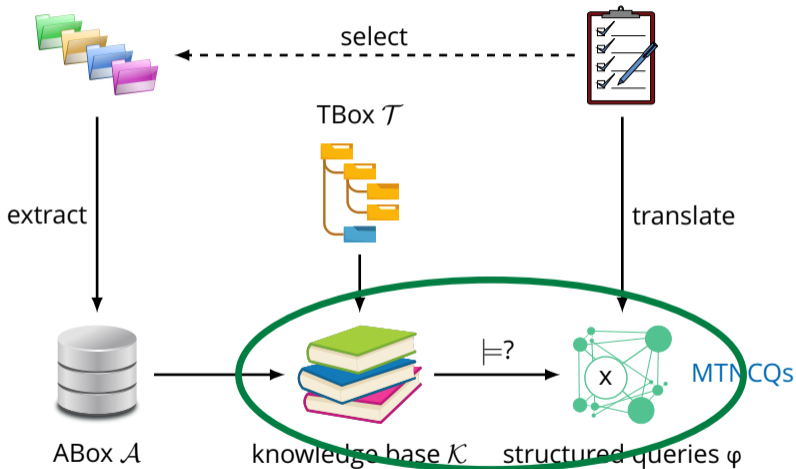
clinical trial criteria



Flashback

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Metric Temporal Conjunctive Queries with Negation (MTNCQs)

“Diagnosis of Rheumatoid Arthritis (RA)
of more than 6 months and less than 15 years.”

<https://clinicaltrials.gov/ct2/show/NCT01198002>

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$$\square_{[-6,0]}(\exists y.\text{diagnosis}(x,y) \wedge \text{RA}(y)) \wedge \neg \square_{[-180,0]}(\exists y.\text{diagnosis}(x,y) \wedge \text{RA}(y))$$

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negative information is rarely entailed \rightsquigarrow use “negation by failure”

Minimal-World Semantics

(Borgwardt and Forkel 2019)

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Knowledge base \mathcal{K} :

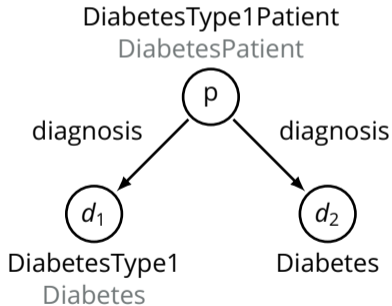
- DiabetesPatient $\equiv \exists \text{diagnosis}.\text{Diabetes}$
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canonical model $\mathcal{I}_{\mathcal{K}}$:



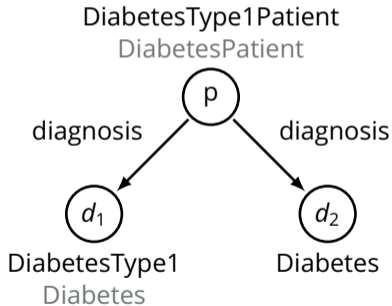
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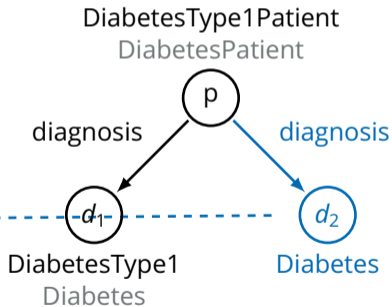
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Patient p has a diagnosis with
Diabetes that is **not** DiabetesType1.



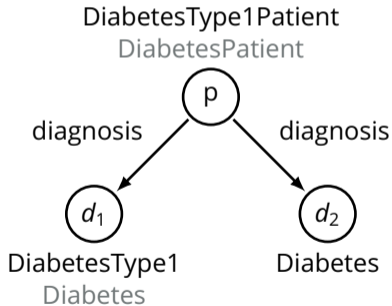
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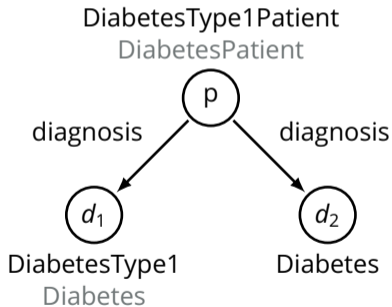
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Minimal-world semantics:
use $\mathcal{I}_{\mathcal{K}} \models \varphi$ instead of $\mathcal{K} \models \varphi$

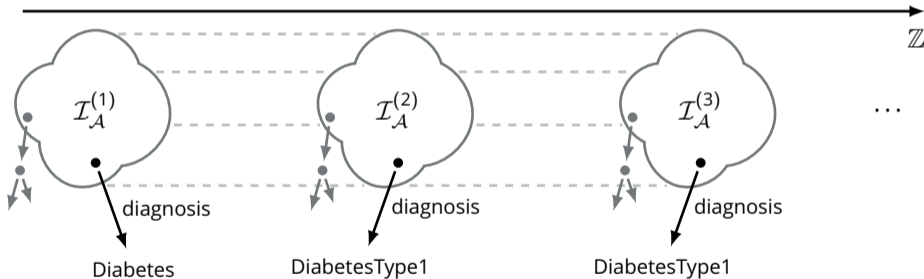


Temporal Minimal-World Semantics

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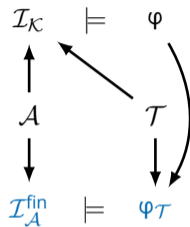
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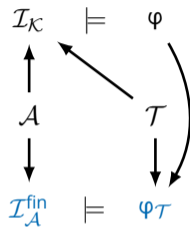
Combined Rewriting

$\mathcal{I}_{\mathcal{K}}$ can be infinite!



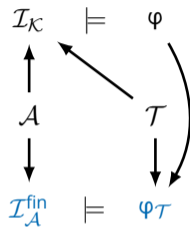
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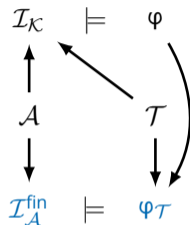


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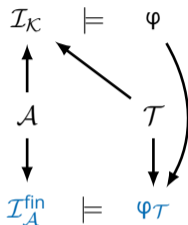


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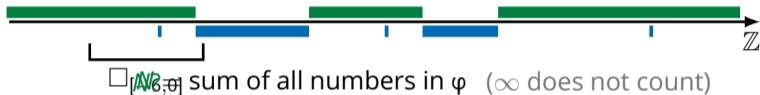


Theorem

In $\mathcal{TECH}_{\perp}^{\diamond, \text{lhs}}$, entailment of rooted MTNCQs is EXPSPACE-complete, and P-complete in data complexity.

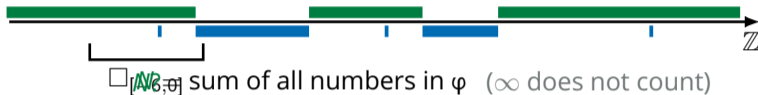
It's Not That Easy

Queries can distinguish more time points!



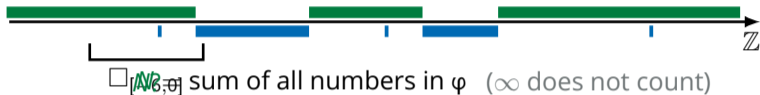
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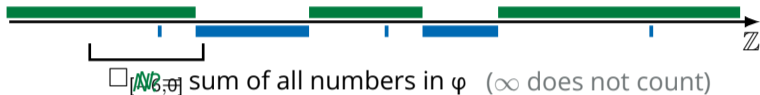
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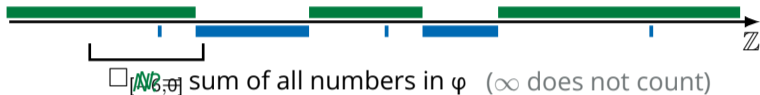
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$$\begin{array}{ccc} \mathcal{A} & \mathcal{T} & \varphi \\ \downarrow & \downarrow & \swarrow \\ \mathcal{I}_{\mathcal{A}}^{\text{fin}} & \models [\varphi_{\mathcal{T}}]^n(t) & \text{quantify over } t+n, \text{ where } n \in [-N, N] \end{array}$$

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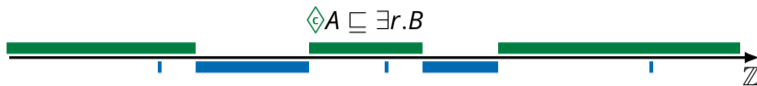
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N , and therefore $\varphi_{\mathcal{T}}$, is exponentially large.

Model checking in FOL is in PSPACE.

→ EXPSPACE

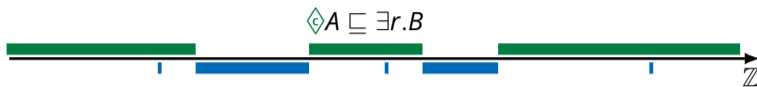
Summary



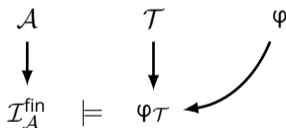
$$\diamond A \sqsubseteq \exists r.B$$

Entailment in $\mathcal{TECH}_{\perp}^{\diamond, \text{lhs}}$ over sparse data is P-complete.

Summary

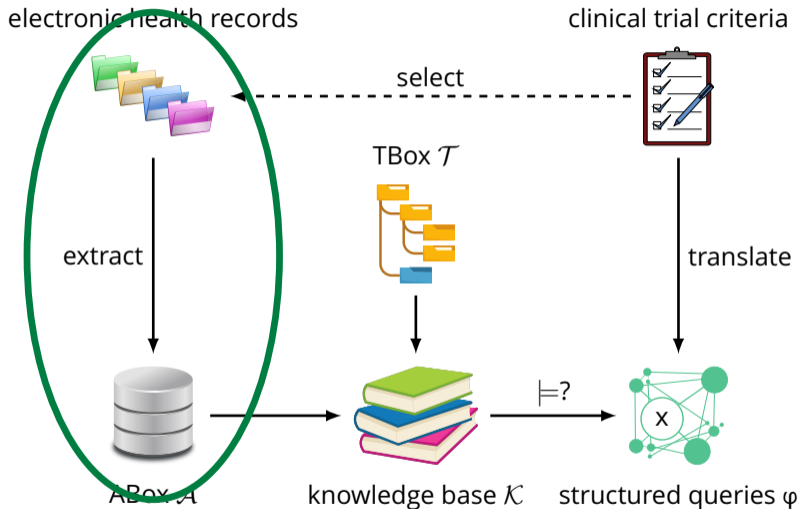


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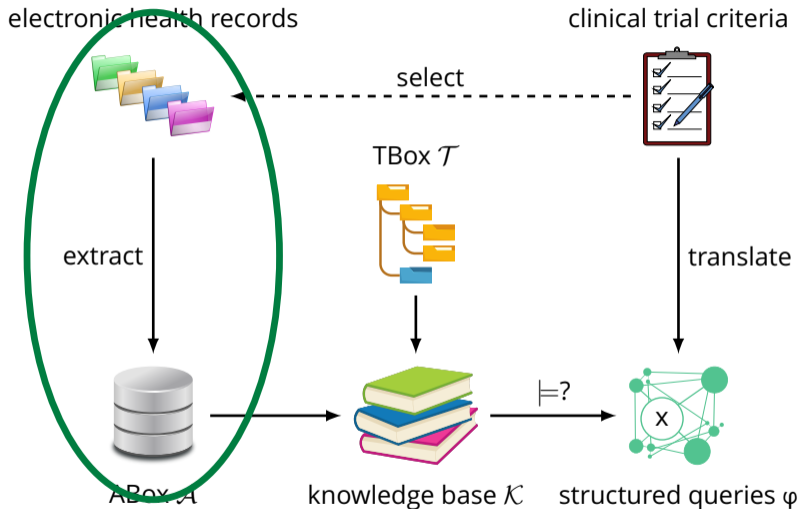


Answering MTNCQs in $\mathcal{TECH}_{\perp}^{\diamond, \text{lhs}}$ is P-complete in data complexity.

Outlook



Outlook



Thank you!

Borgwardt, Stefan and Walter Forkel (2019). "Closed-World Semantics for Conjunctive Queries with Negation over \mathcal{ELH}_\perp Ontologies". In: *Proc. JELIA'19*, pages 371–386.

Gutiérrez-Basulto, Víctor, Jean Christoph Jung, and Roman Kontchakov (2016). "Temporalized \mathcal{EL} Ontologies for Accessing Temporal Data: Complexity of Atomic Queries". In: *Proc. IJCAI'16*, pages 1102–1108.

Pictures from <https://publicdomainvectors.org>