TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

Institute of Theoretical Computer Science Chair of Automata Theory

RECENT ADVANCES IN UNIFICATION FOR THE $\mathcal{EL}$ FAMILY

Franz Baader **Stefan Borgwardt** Barbara Morawska

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The Description Logics $\mathcal{EL}$ and $\mathcal{ELH}_{R+}$

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axioms

GCI	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
transitivity axiom	$r \circ r \sqsubseteq r$	$r^{\mathcal{I}} \circ r^{\mathcal{I}} \subseteq r^{\mathcal{I}}$
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consequences of an $\mathcal{ELH}_{R+}$-ontology $\mathcal{O}$ (finite set of axioms)

subsumption	$C \sqsubseteq_{\mathcal{O}} D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$	(in all models of $\mathcal{O}$)
equivalence	$C \equiv_{\mathcal{O}} D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$	

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Subsumption can be checked in polynomial time. [Baader, Brandt, Lutz IJCAI'05]

If $\mathcal{O}$ contains only GCIs, we call it an $\mathcal{EL}$-ontology.

Unification in $\mathcal{EL}$

Some concept names are variables ($X \in \mathsf{N_v}$), all others are constants ($A \in \mathsf{N_c}$).

$\rightarrow$ unification problem: $\Gamma = \{C_1 \equiv^? D_1, \ldots, C_n \equiv^? D_n\}$
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A unifier σ (w.r.t. $\mathcal{O}$) substitutes variables with concept terms such that

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Unification modulo the theory of bounded semilattices with monotone operators:

A	$\rightarrow$	free constant
X	$\rightarrow$	variable
$\sqcap$	$\rightarrow$	binary associative, commutative, idempotent operator
$\exists r.$	$\rightarrow$	unary monotone operator
$\top$	$\rightarrow$	constant; unit for $\sqcap$
$C \sqsubseteq D$	$\rightarrow$	ground identity
$r \circ r \sqsubseteq r$	$\rightarrow$	non-ground identity ($\exists r.\exists r.X \sqsubseteq \exists r.X$)

Why only ground ontologies?

$$C \equiv_{\mathcal{O}} D \text{ iff } t_C =_{SLmO \cup G_{\mathcal{O},\mathcal{EL}} \cup E_{\mathcal{O},\mathcal{H}_{R+}}} t_D$$

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$\mathcal{O}$ ground: *SLmO*-unification with additional identities:

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$\mathcal{O}$ not ground: Rigid *G*-unification with background theory *SLmO* and additional identities:

$$\sigma(s_i) =_{SLmO \cup \sigma(G) \cup E} \sigma(t_i)$$

Simultaneous rigid *E*-unification is undecidable. [Degtyarev, Voronkov 1996]

Results

Unification in $\mathcal{EL}$ w.r.t. $\mathcal{O} = \emptyset$ is NP-complete.

- Matching is NP-hard. [Küsters '01]
- Unification is in NP. [Baader, Morawska RTA'09/LMCS'10/LPAR'10]
- We can restrict the search to local unifiers of polynomial size.

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Unification in $\mathcal{EL}^{-\top}$ w.r.t. $\mathcal{O} = \emptyset$ is PSPACE-complete.
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Unification in $\mathcal{EL}$ w.r.t. $\mathcal{ELH}_{R+}$-ontologies is NP-complete.

- Brute-force algorithm for $\mathcal{EL}$-ontologies [KR'12]
- Rule-based algorithm for $\mathcal{EL}$-ontologies [DL'12]
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Unification in $\mathcal{ALCO}$ and $\mathcal{SHI}$ is undecidable. [Wolter, Zakharyaschev 2008]

Characterization of Subsumption for Ground Concepts

for $\mathcal{O} = \emptyset$:

- $C_1 \sqcap \cdots \sqcap C_n \sqsubseteq_\emptyset D_1 \sqcap \cdots \sqcap D_m$ iff for all D_j there is a C_i with $C_i \sqsubseteq_\emptyset D_j$.

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for $\mathcal{O} \neq \emptyset$:

- more cases for axioms in $\mathcal{O}$
- $A \sqsubseteq_\mathcal{O} B$ with $A, B \in \mathsf{N_C}$ and $A \neq B$ can hold, e.g., if

$$A \sqsubseteq \exists r.C,\ \ C \sqsubseteq D,\ \ \exists s.D \sqsubseteq B,\ \ r \sqsubseteq s$$

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  are in $\mathcal{O}$.
- two different characterizations in [KR'12] (for $\mathcal{EL}$) and [IJCAR'12] (for $\mathcal{ELH}_{R+}$)

Locality and the Brute-Force Approach

flat atom: concept name or existential restriction $\exists r.A$ with $A \in \mathsf{N_C}$

assumption: unification problem Γ and all GCIs in the $\mathcal{ELH}_{R+}$-ontology $\mathcal{O}$ contain only (conjunctions of) flat atoms

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acyclic assignment S:

$$S_X = \{B\} \qquad \sigma_S(X) = B$$

$$S_Y = \{\exists s.A, \exists r.X\} \qquad \sigma_S(Y) = \exists s.A \sqcap \exists r.B$$

↑ local substitution

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Goal: Every unifiable unification problem has a local unifier.

Necessary Restriction

$\mathcal{O}$ is cycle-restricted if

$$C \not\sqsubseteq_{\mathcal{O}} \exists r_1 . \ldots . \exists r_n . C$$

holds for all concept descriptions C and $r_1, \ldots, r_n \in \mathsf{N_R}$, $n \geq 1$.

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This ensures that variables have no cyclic dependencies under unifiers, i.e., that there is no subsumption $\gamma(X) \sqsubseteq_{\mathcal{O}} \exists r_1 \dots \exists r_n.\gamma(X)$ for any substitution γ.

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Cycle-restrictedness can be checked in polynomial time.

Rule-based approach

nondeterministic rules can be applied to a subsumption in order to solve it:

$$\exists r.X \sqcap \exists r.Y \sqsubseteq^? \exists r.A \;\longrightarrow\; X \sqsubseteq^? A$$

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eager rules are always applied first:

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with more GCIs in $\mathcal{O}$, we have more choices:

$$\underset{(C\text{ ground})}{C \sqsubseteq^? \exists r.X} \xrightarrow{\;C \sqsubseteq_{\mathcal{O}} \exists r.B\;} B \sqsubseteq^? X$$

SAT translation

propositional variables $[C \sqsubseteq D]$ for all $C, D \in \mathrm{At}_{\mathrm{tr}}$

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+ clauses asserting the subsumptions in the unification problem

+ clauses forcing acyclicity of the induced assignment S

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Both algorithms express the unification problem using the characterization of subsumption.

Soundness: acyclicity of S, soundness of the characterization

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$\mathcal{X} \subseteq \mathsf{N_v}$: $\sigma \succ_{\mathcal{X}} \gamma$ iff $\sigma(X) \sqsubseteq_{\mathcal{O}} \gamma(X)$ for all $X \in \mathcal{X}$

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These algorithms cannot be modified to yield exactly the $\succ_{\mathcal{X}}$-minimal unifiers while staying in NP.

[AiML'12]

Conclusions

Summary:

- three NP-algorithms for $\mathcal{ELH}_{R+}$-unification w.r.t. cycle-restricted ontologies
- $\succ_{N_v}$-minimal unifiers are enough, but more difficult to find
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Future Work:

- extend the implementation to cycle-restricted $\mathcal{ELH}_{R+}$-ontologies
- general ontologies? fixpoint semantics for cyclic assignments?
- extensions of $\mathcal{ELH}_{R+}$ ($\mathcal{EL}^{+}$, $\mathcal{FLE}$, $\mathcal{ALC}$)?

Thank You

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