



RECENT ADVANCES IN UNIFICATION FOR THE \mathcal{EL} FAMILY

Franz Baader **Stefan Borgwardt** Barbara Morawska

Manchester, July 1, 2012

The Description Logics \mathcal{EL} and \mathcal{ELH}_{R+}

Syntax

concept name

$A \in N_C$

role name

$r \in N_R$

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consequences of an \mathcal{ELH}_{R+} -ontology \mathcal{O} (finite set of axioms)

subsumption	$C \sqsubseteq_{\mathcal{O}} D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$	(in all models of \mathcal{O})
equivalence	$C \equiv_{\mathcal{O}} D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$	

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Subsumption can be checked in polynomial time. [Baader, Brandt, Lutz IJCAI'05]

If \mathcal{O} contains only GCIs, we call it an \mathcal{EL} -ontology.

Unification in \mathcal{EL}

Some concept names are variables ($X \in N_v$), all others are constants ($A \in N_c$).

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A **unifier** σ (w.r.t. \mathcal{O}) substitutes variables with concept terms such that

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Unification modulo the theory of **bounded semilattices with monotone operators**:

A	→	free constant
X	→	variable
\sqcap	→	binary associative, commutative, idempotent operator
$\exists r.$	→	unary monotone operator
\top	→	constant; unit for \sqcap
$C \sqsubseteq D$	→	ground identity
$r \circ r \sqsubseteq r$	→	non-ground identity ($\exists r. \exists r. X \sqsubseteq \exists r. X$)

Why only ground ontologies?

$$C \equiv_{\mathcal{O}} D \text{ iff } t_C = SLM \cup G_{\mathcal{O}, \mathcal{EL}} \cup E_{\mathcal{O}, \mathcal{H}_R^+} t_D$$

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$$C \equiv_{\mathcal{O}} D \text{ iff } t_C =_{SLmO \cup G_{\mathcal{O}, \mathcal{E}\mathcal{L}} \cup E_{\mathcal{O}, \mathcal{H}_{R+}}} t_D$$

\mathcal{O} ground: *SLmO*-unification with additional identities:

$$\sigma(s_i) =_{SLmO \cup G \cup E} \sigma(t_i)$$

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\mathcal{O} ground: $SLmO$ -unification with additional identities:

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\mathcal{O} not ground: Rigid G -unification with background theory $SLmO$ and additional identities:

$$\sigma(s_i) =_{SLmO \cup \sigma(G) \cup E} \sigma(t_i)$$

Simultaneous rigid E -unification is undecidable. [Degtyarev, Voronkov 1996]

Results

Unification in \mathcal{EL} w.r.t. $\mathcal{O} = \emptyset$ is **NP-complete**.

- Matching is NP-hard. [Küsters '01]
- Unification is in NP. [Baader, Morawska RTA'09/LMCS'10/LPAR'10]
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Unification in $\mathcal{EL}^{-\top}$ w.r.t. $\mathcal{O} = \emptyset$ is **PSPACE-complete**.

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Unification in \mathcal{EL} w.r.t. \mathcal{ELH}_{R+} -ontologies is **NP-complete**.

- Brute-force algorithm for \mathcal{EL} -ontologies [KR'12]
- Rule-based algorithm for \mathcal{EL} -ontologies [DL'12]
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Unification in \mathcal{ALCO} and \mathcal{SHI} is **undecidable**. [Wolter, Zakharyashev 2008]

Characterization of Subsumption for Ground Concepts

for $\mathcal{O} = \emptyset$:

- $C_1 \sqcap \dots \sqcap C_n \sqsubseteq_{\emptyset} D_1 \sqcap \dots \sqcap D_m$ iff for all D_j there is a C_i with $C_i \sqsubseteq_{\emptyset} D_j$.

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for $\mathcal{O} \neq \emptyset$:

- more cases for axioms in \mathcal{O}
- $A \sqsubseteq_{\mathcal{O}} B$ with $A, B \in N_C$ and $A \neq B$ can hold, e.g., if

$$A \sqsubseteq \exists r.C, C \sqsubseteq D, \exists s.D \sqsubseteq B, r \sqsubseteq s$$

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- two different characterizations in [KR'12] (for \mathcal{EL}) and [IJCAR'12] (for \mathcal{ELH}_{R+})

Locality and the Brute-Force Approach

flat atom: concept name or existential restriction $\exists r.A$ with $A \in N_C$

assumption: unification problem Γ and all GCIs in the \mathcal{ELH}_{R^+} -ontology \mathcal{O} contain only (conjunctions of) flat atoms

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acyclic assignment S :

$$S_X = \{B\}$$

$$S_Y = \{\exists s.A, \exists r.X\}$$

$$\sigma_S(X) = B$$

$$\sigma_S(Y) = \exists s.A \sqcap \exists r.B$$

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local substitution

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Goal: Every unifiable unification problem has a local unifier.

Necessary Restriction

\mathcal{O} is **cycle-restricted** if

$$C \not\sqsubseteq_{\mathcal{O}} \exists r_1 \dots \exists r_n . C$$

holds for all concept descriptions C and $r_1, \dots, r_n \in \mathbf{N}_R$, $n \geq 1$.

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This ensures that variables have no cyclic dependencies under unifiers, i.e., that there is no subsumption $\gamma(X) \sqsubseteq_{\mathcal{O}} \exists r_1 \dots \exists r_n. \gamma(X)$ for any substitution γ .

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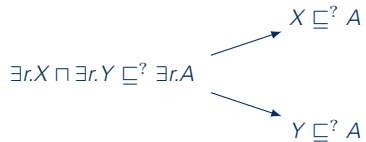
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Cycle-restrictedness can be checked in polynomial time.

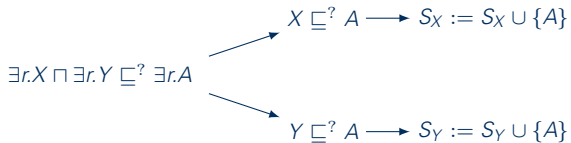
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$$\begin{array}{l} \exists r.X \sqcap \exists r.Y \sqsubseteq^? \exists r.A \\ \swarrow \quad \searrow \\ X \sqsubseteq^? A \longrightarrow S_X := S_X \cup \{A\} \\ Y \sqsubseteq^? A \longrightarrow S_Y := S_Y \cup \{A\} \end{array}$$

eager rules are always applied first:

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with more GCIs in \mathcal{O} , we have more choices:

$$\begin{array}{l} C \sqsubseteq^? \exists r.X \\ \text{(C ground)} \end{array} \xrightarrow{C \sqsubseteq_{\mathcal{O}} \exists r.B} B \sqsubseteq^? X$$

SAT translation

propositional variables $[C \sqsubseteq D]$ for all $C, D \in \text{At}_{\text{tr}}$

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+ clauses forcing acyclicity of the induced assignment S

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Both algorithms express the unification problem using the characterization of subsumption.

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These algorithms cannot be modified to yield exactly the $\succ_{\mathcal{X}}$ -minimal unifiers while staying in NP.

[AiML'12]

Conclusions

Summary:

- three NP-algorithms for \mathcal{ELH}_{R+} -unification w.r.t. cycle-restricted ontologies
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Future Work:

- extend the implementation to cycle-restricted \mathcal{ELH}_{R+} -ontologies
- general ontologies? fixpoint semantics for cyclic assignments?
- extensions of \mathcal{ELH}_{R+} (\mathcal{EL}^+ , \mathcal{FLE} , \mathcal{ALC})?

Thank You



Franz Baader, Stefan Borgwardt, and Barbara Morawska.
Computing minimal \mathcal{EL} -unifiers is hard.
In *Advances in Modal Logic 9 (AiML'12)*. College Publications, 2012.
To appear.



Franz Baader, Stefan Borgwardt, and Barbara Morawska.
Extending unification in \mathcal{EL} towards general TBoxes.
In *Proc. KR'12*. AAAI Press, 2012.



Franz Baader, Stefan Borgwardt, and Barbara Morawska.
SAT-encoding of unification in \mathcal{ELH}_{R+} w.r.t. cycle-restricted ontologies.
In *Proc. IJCAR'12*, volume 7364 of *LNAI*, pages 30–44. Springer, 2012.



Anatoli Degtyarev and Andrei Voronkov.
The undecidability of simultaneous rigid E -unification.
Theor. Comput. Sci., 166(1–2):291–300, 1996.



Frank Wolter and Michael Zakharyashev.
Undecidability of the unification and admissibility problems for modal and description logics.
ACM T. Comput. Log., 9(4), 2008.