Concise Justifications Versus Detailed Proofs for Description Logic Entailments

or: Does Size Matter? // XLoKR workshop, 5th November, 2021
Explanations for Description Logics (DLs)

- DLs are used in large ontologies (Galen, SNOMED CT, ...)
- How to explain $\text{NervousSystemSign} \sqsubseteq \text{AnatomicalSystemSign}$?

Concise Justifications Versus Detailed Proofs
Technische Universität Dresden // © Stefan Borgwardt
XLoKR workshop, 5th November, 2021
Explanations for Description Logics (DLs)

- DLs are used in large ontologies (Galen, SNOMED CT, …)
- How to explain $\text{NervousSystemSign} \sqsubseteq \text{AnatomicalSystemSign}$?
- Two approaches: justifications vs. proofs
- Proofs can be computed from justifications
Explanations for Description Logics (DLs)

- DLs are used in large ontologies (Galen, SNOMED CT, …)
- How to explain $\text{NervousSystemSign} \subseteq \text{AnatomicalSystemSign}$?
- Two approaches: justifications vs. proofs
- Proofs can be computed from justifications
- Here: empirical comparison, e.g., size of justifications vs. size of proofs
- Practical relevance: compute justification first, then proof from justification
Justifications

minimal subsets entailing a given consequence
- usually small
- inferences may be not obvious
- often several justifications
Proofs

trees/graphs containing multiple inference steps
- can be large
- individual steps often obvious
- contain justifications (as leaves)
- often several proofs

Concise Justifications Versus Detailed Proofs
Technische Universität Dresden // © Stefan Borgwardt
XLoKR workshop, 5th November, 2021
Proofs

trees/graphs containing multiple inference steps

- can be large
- individual steps often obvious
- contain justifications (as leaves)
- often several proofs

small justification \(\Rightarrow\) small proof?
Flashback: XLoKR 2020

Christian Alrabbaa, Stefan Borgwardt, Patrick Koopmann, and Alisa Kovtunova: Finding Proofs for Description Logic Entailments in Practice

- Corpus of proofs extracted from standard benchmark
- Comparison of different ways to compute proofs
Flashback: XLoKR 2020

Christian Alrabbaa, Stefan Borgwardt, Patrick Koopmann, and Alisa Kovtunova: Finding Proofs for Description Logic Entailments in Practice

- Corpus of proofs extracted from standard benchmark
- Comparison of different ways to compute proofs

And now some slides from last year …
Proofs with ELK

- ELK uses a *consequence-based* reasoning procedure

⇒ inferences performed using a calculus

\[
\begin{align*}
R_0 & \quad \frac{C \sqsubseteq C}{ } \\
R_T & \quad \frac{C \sqsubseteq T}{ } \\
R_n & \quad \frac{C \sqsubseteq D \cap E}{C \sqsubseteq D} \\
R_\exists & \quad \frac{C \sqsubseteq \exists r.D}{D \sqsubseteq E} \\
R_\perp & \quad \frac{C \sqsubseteq \exists r.D}{D \sqsubseteq \bot} \\
R_\circ & \quad \frac{C_0 \sqsubseteq \exists r_1.C_1, \ldots, C_{n-1} \sqsubseteq \exists r_n.C_n}{C_0 \sqsubseteq \exists r.C_n} : r_1 \circ \ldots \circ r_n \sqsubseteq r \in \mathcal{O}
\end{align*}
\]

⇒ proofs generated as part of reasoning process
Forgetting-Based Proofs

- Idea from propositional resolution:

\[
\frac{q_1 \lor p \quad q_2 \lor \neg p}{q_1 \lor q_2}
\]

⇒ inference through elimination of \( p \)
Forgetting-Based Proofs

- Idea from propositional resolution:

\[
\begin{align*}
q_1 \lor p & \quad q_2 \lor \neg p \\
\hline
q_1 \lor q_2
\end{align*}
\]

⇒ inference through elimination of \( p \)

- Idea: Use similar approach to prove axioms of the form \( A \sqsubseteq B \)

\[
A \sqsubseteq C \quad C \sqsubseteq \exists r.D \quad \exists r.\top \sqsubseteq B
\]
Forgetting-Based Proofs

- Idea from propositional resolution:

\[
\begin{array}{c}
q_1 \lor p \\
q_2 \lor \neg p
\end{array}
\Rightarrow
\begin{array}{c}
q_1 \lor q_2
\end{array}
\]

- inference through elimination of \( p \)

- Idea: Use similar approach to prove axioms of the form \( A \sqsubseteq B \)

\[
\begin{align*}
A & \sqsubseteq C \\
C & \sqsubseteq \exists r.D \\
\exists r. \top & \sqsubseteq B
\end{align*}
\]
Forgetting-Based Proofs

- Idea from propositional resolution:
  \[ q_1 \lor p \quad q_2 \lor \neg p \]
  \[ \Rightarrow q_1 \lor q_2 \]

\[ \Rightarrow \text{inference through elimination of } p \]

- Idea: Use similar approach to prove axioms of the form \( A \sqsubseteq B \)

\[
\begin{align*}
A \sqsubseteq C & \quad C \sqsubseteq \exists r. \top & \exists r. \top \sqsubseteq B
\end{align*}
\]
Forgetting-Based Proofs

- Idea from propositional resolution:
  \[ q_1 \lor p \quad q_2 \lor \neg p \]
  \[ \frac{q_1 \lor \neg p}{q_1 \lor q_2} \]
  ⇒ inference through elimination of \( p \)

- Idea: Use similar approach to prove axioms of the form \( A \sqsubseteq B \)
  \[ A \sqsubseteq C \quad C \sqsubseteq \exists r. T \quad \exists r. T \sqsubseteq B \]
Forgetting-Based Proofs

- Idea from propositional resolution:
  \[
  \frac{q_1 \lor p \quad q_2 \lor \neg p}{q_1 \lor q_2}
  \]

  \[\Rightarrow\] inference through elimination of \( p \)

- Idea: Use similar approach to prove axioms of the form \( A \sqsubseteq B \)
  
  \[
  \begin{align*}
  A & \sqsubseteq C \quad C \sqsubseteq B \\
  \end{align*}
  \]
Forgetting-Based Proofs

• Idea from propositional resolution:
  \[
  \frac{q_1 \lor p \quad q_2 \lor \neg p}{q_1 \lor q_2}
  \]
  \Rightarrow \text{inference through elimination of } p

• Idea: Use similar approach to prove axioms of the form \( A \sqsubseteq B \)

\[
A \sqsubseteq C \quad C \sqsubseteq B
\]
Forgetting-Based Proofs

- Idea from propositional resolution:
  \[
  \frac{q_1 \lor p \quad q_2 \lor \neg p}{q_1 \lor q_2}
  \]
  \[\Rightarrow\] inference through elimination of \(p\)

- Idea: Use similar approach to prove axioms of the form \(A \sqsubseteq B\)

  \[
  A \sqsubseteq B
  \]

\[\text{Concise Justifications Versus Detailed Proofs}\]
\[\text{Technische Universität Dresden // © Stefan Borgwardt}\]
\[\text{XLoKR workshop, 5th November, 2021}\]
Forgetting-Based Proofs

• Idea from propositional resolution:

\[
\frac{q_1 \lor p \quad q_2 \lor \neg p}{q_1 \lor q_2}
\]

⇒ inference through elimination of \( p \)

• Idea: Use similar approach to prove axioms of the form \( A \sqsubseteq B \)

\[
\begin{align*}
A \sqsubseteq C \\
C \sqsubseteq \exists r. D \\
\exists r. T \sqsubseteq B
\end{align*}
\]

\[
\frac{A \sqsubseteq C \\
\frac{C \sqsubseteq \exists r. D}{C \sqsubseteq \exists r. T} \\
\exists r. T \sqsubseteq B}
\]

\[
\frac{A \sqsubseteq \exists r. T}{A \sqsubseteq B}
\]
Evaluation

Proof generators
1. Smallest proofs generated by ELK calculus
2. Forgetting-based proofs using $\text{ALCH}$ variant of LETHE 0.6
3. Forgetting-based proofs using $\text{ALCOI}$ variant of FAME 1.0

Corpus
- $\text{ELH}$ Ontologies from OWL Reasoner Evaluation 2015, OWL EL Classification Track
  - well-balanced mix of ontologies from different repositories
- Extracted 1,573 justification patterns
  - all entailments of the form $A \sqsubseteq B$ or $A \equiv B$
  - all justifications for these entailments modulo renaming of symbols

Metrics
1. Size (number of axioms)
2. Complexity
  - Horridge et al. 2013: “Toward cognitive support for OWL justifications”
  - Compute maximum and average over all proof steps
Justifications vs. Proofs

... and now we resume our regular programming.
Justifications vs. Proofs

... and now we resume our regular programming.

We first extracted single justifications, then produced (small) proofs. Would a larger justification have allowed us to find a smaller proof?
Justifications vs. Proofs

... and now we resume our regular programming.

We first extracted single justifications, then produced (small) proofs. Would a larger justification have allowed us to find a smaller proof?

Compare each justification (pattern) with the induced proof(s)

1. Justification size vs. proof size
2. Justification complexity vs. average step complexity
3. Justification complexity vs. maximum step complexity
Does Size Matter?

Distribution

Histogram of ratio

Adjusted histogram

Very strong correlation: $r > 0.8$ ($r > 0.95$ after adjustment)
Does Complexity Matter?

Moderate correlation: $r < 0.65$ ($r < 0.5$ after adjustment)
Does Complexity Matter? (II)

Strong correlation: \( r > 0.5 \) (\( r > 0.65 \) after adjustment)
Justification for $A \equiv B$:
\[
\{ A \equiv C, \ C \equiv E, \ E \equiv F, \ D \equiv F, \ B \equiv D \}
\]
(complexity 150)

Proof size: 24 (ratio 4.8)

Average step complexity: 197 (ratio 1.3)

Maximum step complexity: 260 (ratio 1.7)
Extreme Examples (II)

Justification size 14, complexity 900
Proof size: 29 (ratio 2.07)
Average step complexity: 196 (ratio 0.22)
Maximum step complexity: 280 (ratio 0.32)
Summary

- Small justifications (generally) yield small proofs
- Average complexity (generally) goes down
- Maximum complexity goes down only slightly, if at all
Summary

- Small justifications (generally) yield small proofs
- Average complexity (generally) goes down
- Maximum complexity goes down only slightly, if at all

Ongoing work: condensing proofs:

- Middle ground between too much and too little information
- Needs to be tailored to the specific user (e.g., knowledge about DL and domain)
Summary

- Small justifications (generally) yield small proofs
- Average complexity (generally) goes down
- Maximum complexity goes down only slightly, if at all

Ongoing work: condensing proofs:

- Middle ground between too much and too little information
- Needs to be tailored to the specific user (e.g., knowledge about DL and domain)

Thank you!