TECHNISCHE UNIVERSITÄT DRESDEN

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Concise Justifications Versus Detailed Proofs for Description Logic Entailments

or: Does Size Matter? // XLoKR workshop, 5th November, 2021

Explanations for Description Logics (DLs)

- DLs are used in large ontologies (Galen, SNOMED CT, ...)
- How to explain $\mathsf{NervousSystemSign} \sqsubseteq \mathsf{AnatomicalSystemSign}$?

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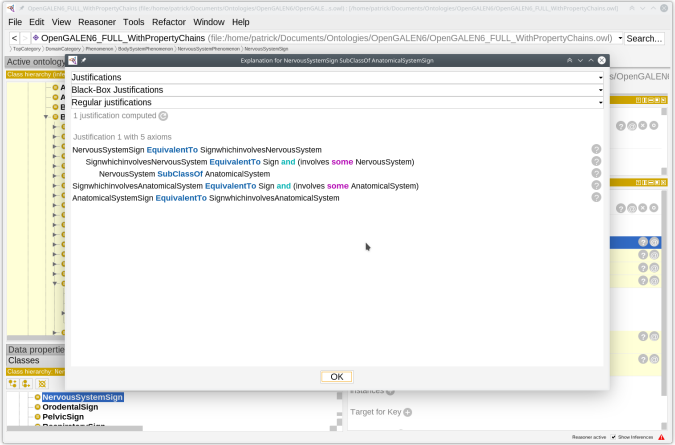
- Two approaches: justifications vs. proofs
- Proofs can be computed from justifications
- Here: empirical comparison, e.g., size of justifications vs. size of proofs
- Practical relevance: compute justification first, then proof from justification

Justifications

minimal subsets entailing a given consequence

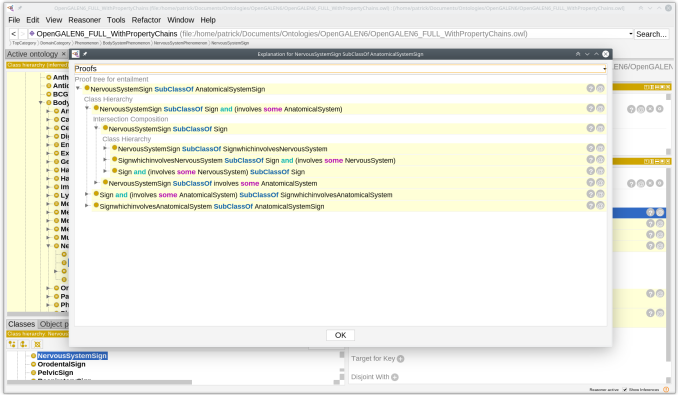
- usually small
- inferences may be not obvious
- often several justifications



Proofs

trees/graphs containing multiple inference steps

- can be large
- individual steps often obvious
- contain justifications (as leaves)
- often several proofs



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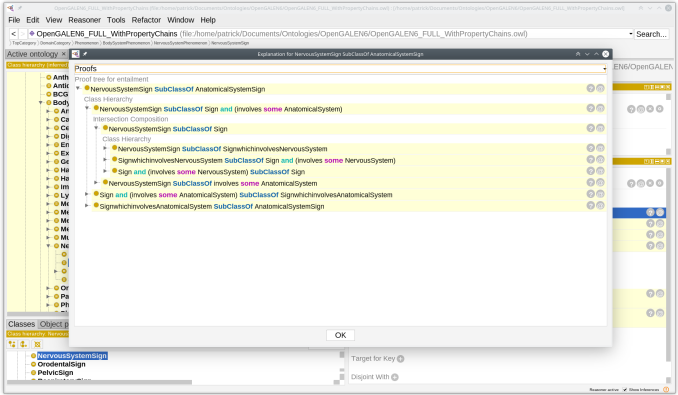
small justification
⇒ small proof?



Flashback: XLoKR 2020

Christian Alrabbaa, Stefan Borgwardt, Patrick Koopmann, and Alisa Kovtunova:
Finding Proofs for Description Logic Entailments in Practice

- Corpus of proofs extracted from standard benchmark
- Comparison of different ways to compute proofs

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And now some slides from last year …

Proofs with ELK

- ELK uses a *consequence-based* reasoning procedure

$\Rightarrow$ inferences performed using a calculus

$$\mathbf{R}_0 \frac{}{C \sqsubseteq C} \qquad \mathbf{R}_\top \frac{}{C \sqsubseteq \top} \qquad \mathbf{R}_\sqcap^- \frac{C \sqsubseteq D \sqcap E}{C \sqsubseteq D \quad C \sqsubseteq E}$$

$$\mathbf{R}_\sqcap^+ \frac{C \sqsubseteq D \quad C \sqsubseteq E}{C \sqsubseteq D \sqcap E} \qquad \mathbf{R}_\exists \frac{C \sqsubseteq \exists r.D \quad D \sqsubseteq E}{C \sqsubseteq \exists r.E}$$

$$\mathbf{R}_\bot \frac{C \sqsubseteq \exists r.D \quad D \sqsubseteq \bot}{C \sqsubseteq \bot} \qquad \mathbf{R}_\sqsubseteq \frac{C \sqsubseteq D}{C \sqsubseteq E} : D \sqsubseteq E \in \mathcal{O}$$

$$\mathbf{R}_\circ \frac{C_0 \sqsubseteq \exists r_1.C_1, C_1 \sqsubseteq \exists r_2.C_2 \ \ldots \ C_{n-1} \sqsubseteq \exists r_n.C_n}{C_0 \sqsubseteq \exists r.C_n} : r_1 \circ \ldots \circ r_n \sqsubseteq r \in \mathcal{O}$$

$\Rightarrow$ proofs generated as part of reasoning process

Forgetting-Based Proofs

- Idea from propositional resolution:

$$\boxed{\frac{q_1 \lor p \qquad q_2 \lor \neg p}{q_1 \lor q_2}}$$

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$$A \sqsubseteq C \qquad C \sqsubseteq \exists r.D \qquad \exists r.\top \sqsubseteq B$$

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$$\dfrac{(C)\ \dfrac{A \sqsubseteq C \qquad (D)\ \dfrac{C \sqsubseteq \exists r.D}{C \sqsubseteq \exists r.\top}}{A \sqsubseteq \exists r.\top}\ (r) \qquad \exists r.\top \sqsubseteq B}{A \sqsubseteq B}$$

Evaluation

Proof generators

1. Smallest proofs generated by ELK calculus
2. Forgetting-based proofs using $\mathcal{ALCH}$ variant of LETHE 0.6
3. Forgetting-based proofs using $\mathcal{ALCOI}$ variant of FAME 1.0

Corpus

- $\mathcal{ELH}$ Ontologies from OWL Reasoner Evaluation 2015, OWL EL Classification Track
  - well-balanced mix of ontologies from different repositories
- Extracted 1,573 *justification patterns*
  - all entailments of the form $A \sqsubseteq B$ or $A \equiv B$
  - all justifications for these entailments modulo renaming of symbols

Metrics

1. Size (number of axioms)
2. Complexity
   - Horridge et al. 2013: "Toward cognitive support for OWL justifications"
   - Compute maximum and average over all proof steps

Justifications vs. Proofs

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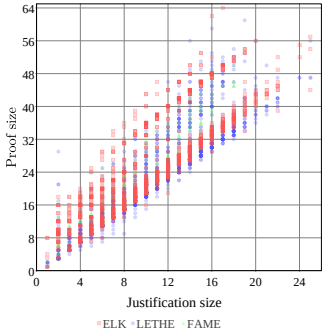
Compare each justification (pattern) with the induced proof(s)

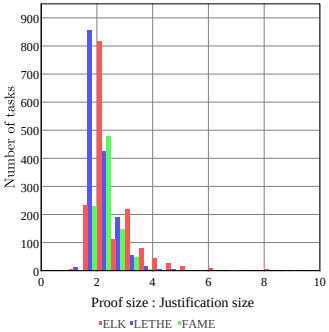
1. Justification size vs. proof size
2. Justification complexity vs. average step complexity
3. Justification complexity vs. maximum step complexity

Does Size Matter?

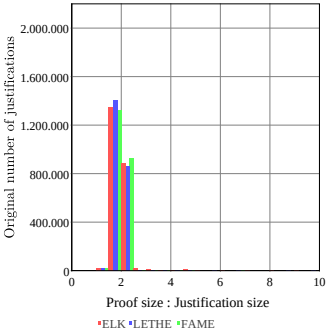
Distribution



Histogram of ratio



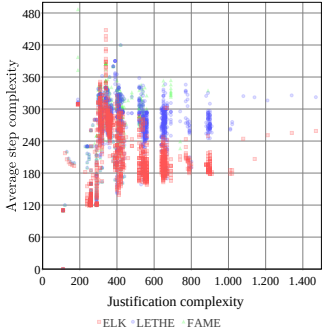
Adjusted histogram



Very strong correlation: $r > 0.8$ ($r > 0.95$ after adjustment)

Does Complexity Matter?

Distribution



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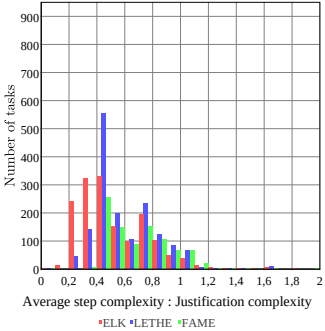

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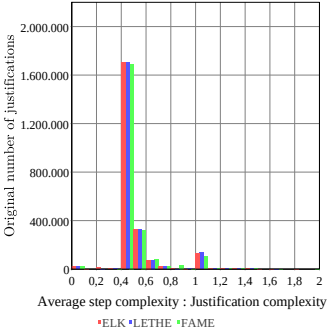
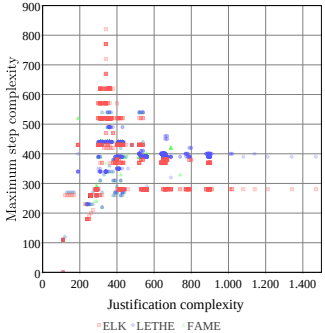


Moderate correlation: $r < 0.65$ ($r < 0.5$ after adjustment)

Does Complexity Matter? (II)

Distribution



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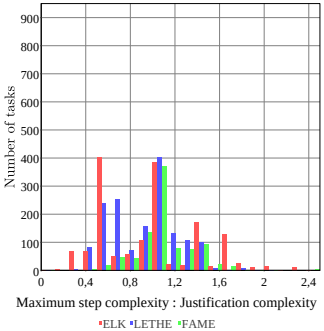

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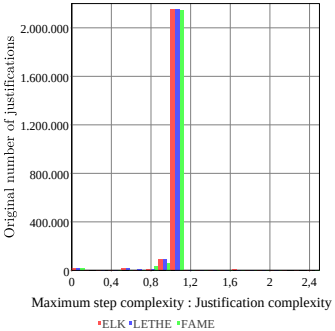
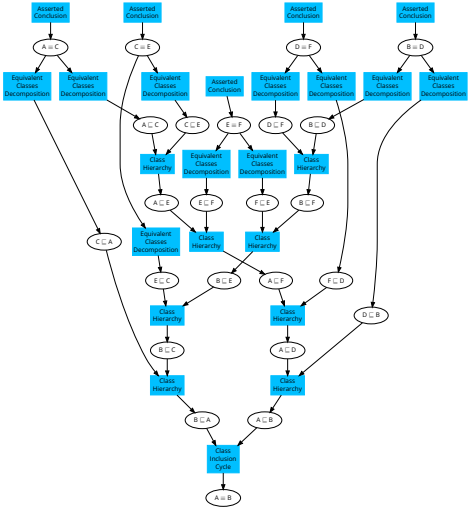


Strong correlation: $r > 0.5$ ($r > 0.65$ after adjustment)

Extreme Examples



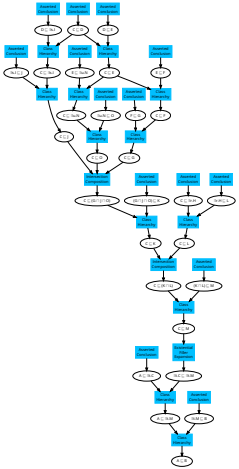
Justification for $A \equiv B$:
$\{A \equiv C,\ C \equiv E,\ E \equiv F,\ D \equiv F,\ B \equiv D\}$
(complexity 150)

Proof size: 24 (ratio 4.8)

Average step complexity: 197 (ratio 1.3)

Maximum step complexity: 260 (ratio 1.7)

Extreme Examples (II)



Justification size 14, complexity 900

Proof size: 29 (ratio 2.07)

Average step complexity: 196 (ratio 0.22)

Maximum step complexity: 280 (ratio 0.32)

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