TECHNISCHE UNIVERSITÄT DRESDEN

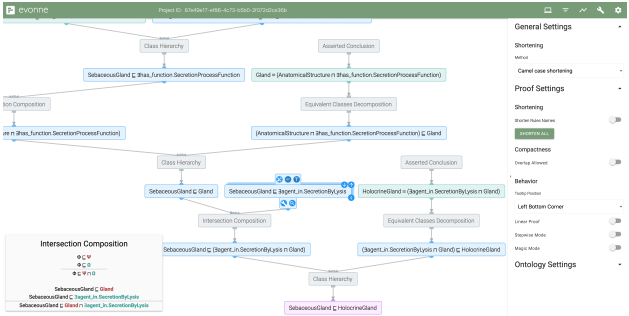
CPEC CENTER FOR PERSPICUOUS COMPUTING

Christian Alrabbaa, Stefan Borgwardt, Patrick Koopmann, Alisa Kovtunova
Technische Universität Dresden, Center for Perspicuous Computing (CPEC)

Finding Good Proofs for Ontology-Mediated Query Answers

(extended abstract of DL paper) // XLoKR'22, Haifa, Israel, 31st July 2022

Motivation



Ontologies, Proofs

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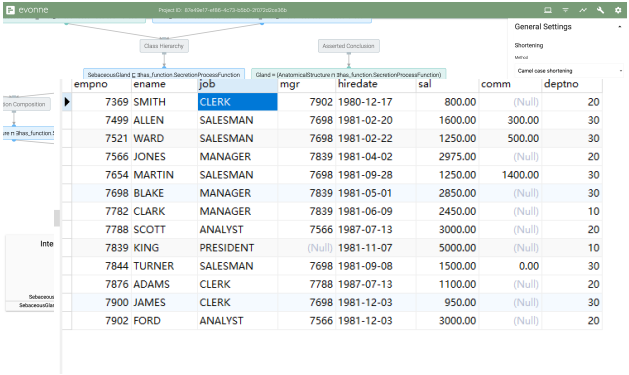

empno	ename	job	mgr	hiredate	sal	comm	deptno
7369	SMITH	CLERK	7902	1980-12-17	800.00	(Null)	20
7499	ALLEN	SALESMAN	7698	1981-02-20	1600.00	300.00	30
7521	WARD	SALESMAN	7698	1981-02-22	1250.00	500.00	30
7566	JONES	MANAGER	7839	1981-04-02	2975.00	(Null)	20
7654	MARTIN	SALESMAN	7698	1981-09-28	1250.00	1400.00	30
7698	BLAKE	MANAGER	7839	1981-05-01	2850.00	(Null)	30
7782	CLARK	MANAGER	7839	1981-06-09	2450.00	(Null)	10
7788	SCOTT	ANALYST	7566	1987-07-13	3000.00	(Null)	20
7839	KING	PRESIDENT	(Null)	1981-11-07	5000.00	(Null)	10
7844	TURNER	SALESMAN	7698	1981-09-08	1500.00	0.00	30
7876	ADAMS	CLERK	7788	1987-07-13	1100.00	(Null)	20
7900	JAMES	CLERK	7698	1981-12-03	950.00	(Null)	30
7902	FORD	ANALYST	7566	1981-12-03	3000.00	(Null)	20

Ontologies, Proofs

plus Data, Queries

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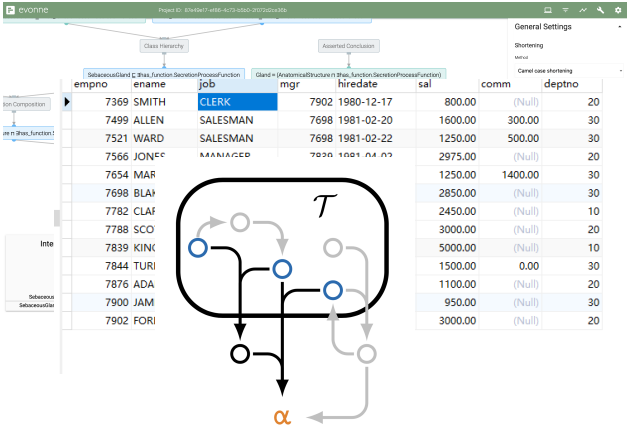

Ontologies, Proofs

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Algorithms, Complexity

Outline

Description Logics (DLs)

DLs are fragments of first-order logic with a funny syntax.

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A DL TBox/ontology $\mathcal{T}$:

existential rules (implicit $\forall$ omitted)	DL syntax
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$\mathrm{partOfW}(x, y) \to \mathrm{Wheel}(y)$	$\exists \mathrm{partOfW}^- \sqsubseteq \mathrm{Wheel}$
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$\mathrm{partOfB}(x, y) \to \mathrm{Bike}(y)$	$\exists \mathrm{partOfB}^- \sqsubseteq \mathrm{Bike}$

Proofs

(Alrabbaa, Baader, Borgwardt, Koopmann, and Kovtunova 2021)

Entailment: $\mathcal{T}$ implies a new sentence α, e.g.,

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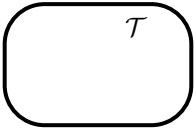

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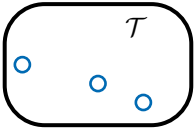

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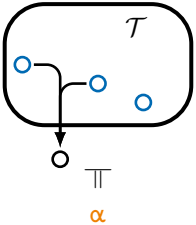

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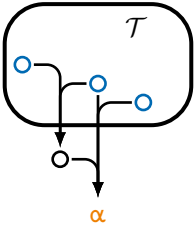

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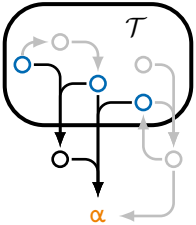

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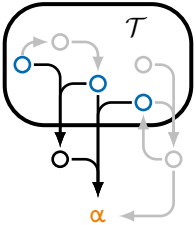

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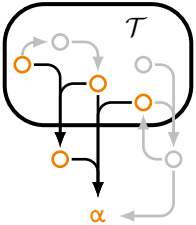

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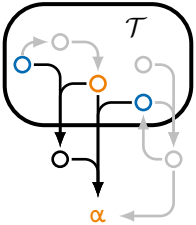

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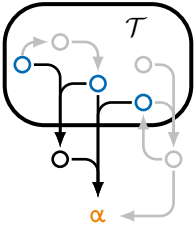

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Decision problem: Decide whether there exists a proof of measure $\leq n$

Ontology-Mediated Queries

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For *DL-Lite*$_R$, this problem is

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What is the complexity of explaining this entailment?

Outline

Deriving CQs ($\mathfrak{D}_{\mathsf{cq}}$)

(Stefanoni 2011; Croce and Lenzerini 2018)

What are the inference rules for proving $\mathcal{T} \cup \mathcal{A} \models q(a)$ (in *DL-Lite*$_R$)?

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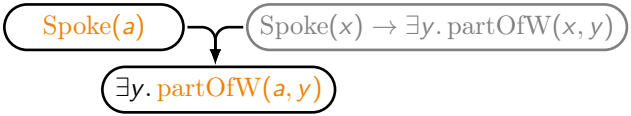

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$$\mathrm{partOfW}(x, y) \to \mathrm{hasPart}(y, x) \quad \exists y, z.\, \mathrm{partOfW}(a, y) \wedge \mathrm{partOfB}(y, z)$$

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Deriving Ground Atoms ($\mathfrak{D}_{\mathsf{sk}}$) (Borgida, Calvanese, and Rodriguez-Muro 2008)

We can use Skolem functions to make proofs more modular and compact.

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$$\mathrm{partOfW}(a, \mathrm{wheel}(a)) \quad \mathrm{pW}(x, y) \to \mathrm{pB}(y, \mathrm{bi}(y))$$

$$\mathrm{partOfB}(\mathrm{wheel}(a), \mathrm{bike}(\mathrm{wheel}(a)))$$

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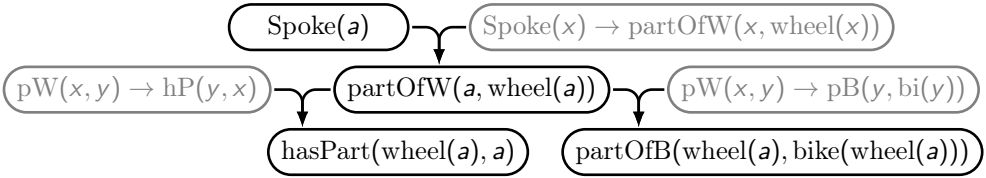
(Borgida, Calvanese, and Rodriguez-Muro 2008)

We can use Skolem functions to make proofs more modular and compact.



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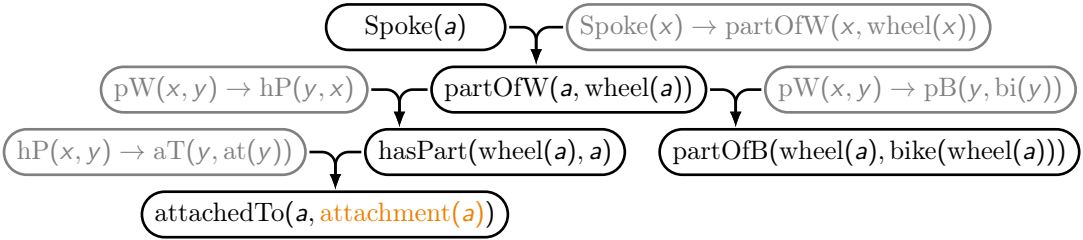

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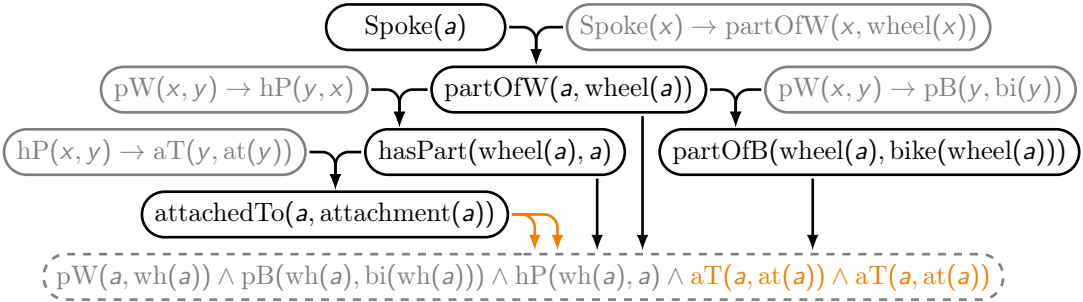

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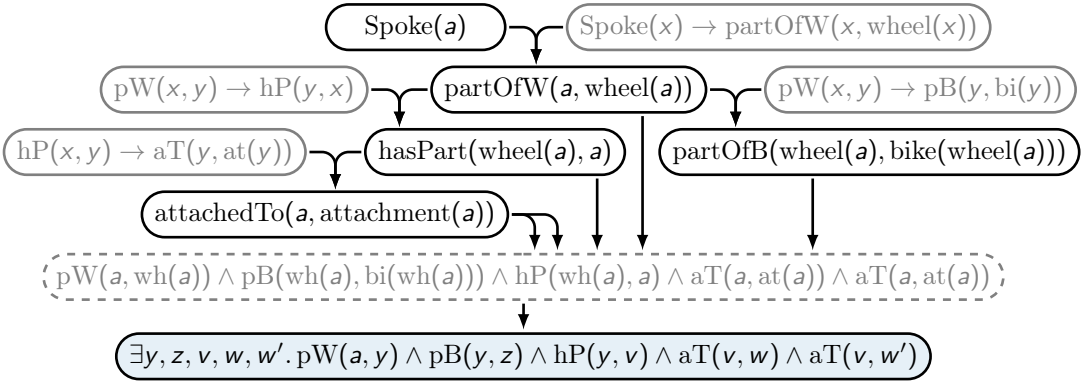

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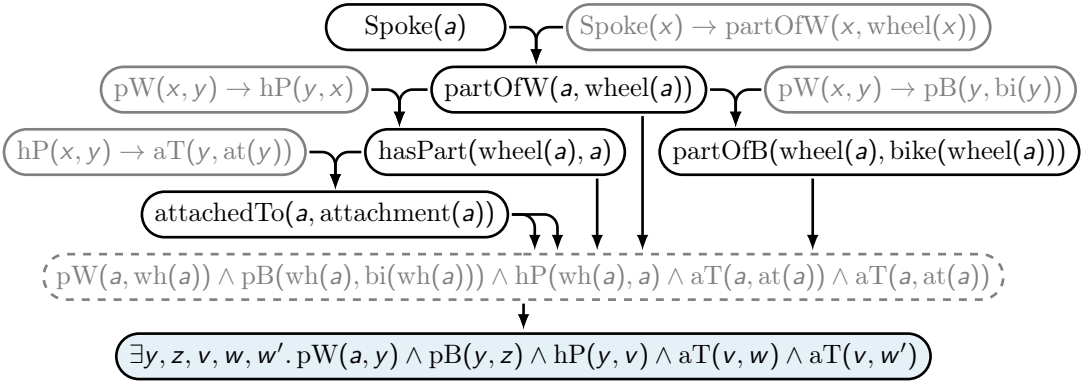

Only sound w.r.t. Skolemized TBox!

Outline

The Complexity of Finding Small (Tree) Proofs

Assuming that $\mathcal{T} \cup \mathcal{A} \models q(a)$ holds, is there a proof of (tree) size $\leq n$ w.r.t. $\mathfrak{D}_{\mathsf{cq}}/\mathfrak{D}_{\mathsf{sk}}$?

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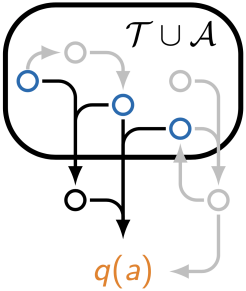
Combined complexity:

- NP-complete for *DL-Lite*$_R$ (tree size can be bounded by a polynomial)
- In P for tree-shaped queries in *DL-Lite*$_R$ w.r.t. tree size and $\mathfrak{D}_{\mathsf{sk}}$ (use placeholders for Skolem terms)
- NP-hard for tree-shaped queries w.r.t. size or $\mathfrak{D}_{\mathsf{cq}}$ and empty TBox (reductions from SAT)

Summary

- Framework for describing proofs of query answers
- Two different kinds of inference rules
- Complexity results for *DL-Lite*$_R$



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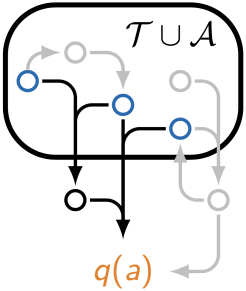

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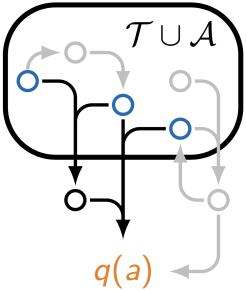

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Thank you!

References I

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