



Introduction to nonmonotonic reasoning

Winter Semester 2019/20

Exercise Sheet 1

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Exercise 1.1 We consider substitutions. The *composition* of two substitutions $\sigma = [X_1/t_1, \dots, X_n/t_n]$ and $\rho = [Y_1/s_1, \dots, Y_m/s_m]$ is defined as follows

$$\sigma \circ \rho = \{X_i/t_i\rho \mid X_i \neq t_i\rho \text{ for } 1 \leq i \leq n\} \cup \{Y_j/s_j \mid Y_j \notin \{X_1, \dots, X_n\} \text{ for } 1 \leq j \leq m\}.$$

- (a) Given the substitutions $\sigma = [V_1/p(), V_3/t(p', p'')]]$ and $\rho = [V_2/V_3, V_4/f_2(V_3, p()), V_5/V_2]$ and the formulae:

$$\varphi_a = \exists V_1 (t_1(V_1, p_1(V_2)) \wedge t_2(f(V_4, p()), V_5))$$

$$\varphi_b = (t_1(V_1, p_1(V_2)) \wedge t_2(f(V_4, p()), V_4))$$

Is $\sigma \circ \rho$ a ground substitution?

- (b) Is $\varphi_a(\sigma \circ \rho)$ a ground formula? Is $\varphi_b(\sigma \circ \rho)$ a ground formula?
(c) Show that substitutions are closed under composition.

Exercise 1.2 "For any formula φ and admissible substitution σ , the formula $\forall X\varphi \rightarrow (\varphi\sigma)$ is a tautology." Does this claim hold or not? Prove or refute the claim.

Exercise 1.3 We are turning to Default logic. Devise a default theory that models the bike shop domain.