Introduction to nonmonotonic reasoning

Winter Semester 2019/20

Exercise Sheet 9 – Inference relations

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Exercise 9.1 Generalize circumscription of p in φ by ψ_p (Definition 5.5) to *finite sets* of formulae $\{\varphi_1, \ldots, \varphi_m\}$.

Exercise 9.2 Show the intermediate claim used in the proof of Theorem 5.7:

$$\mathcal{I} \models_{sta} T[P_1/\psi_1, \ldots, p_k/\psi_k] \iff \mathcal{J} \models_{sta} T.$$

Exercise 9.3 Let T be a first-order theory consisting of the formulae:

$$\forall X_1 \forall X_2 \big(p(X_1, X_2) \longrightarrow \exists X_3 p(X_2, X_3)\big) \quad (1)$$

$$\exists X_2 \exists X_3 \big(X_2, X_3) \land \forall X_1 \neg p(X_1, X_2)\big) \quad (2)$$

$$\forall X_1 \forall X_2 \forall X_3 \big(p(X_2, X_1) \land p(X_3, X_1) \longrightarrow X_3 = X_1\big) \quad (3)$$

Consider the circumscription of p using the predicate expression:

$$\psi_p(X_1, X_2) \equiv \big(p(X_1, X_2) \land \exists X_3 p(X_3, X_1)\big).$$

(a) Verify that all three formulae in $T[p/\psi_p]$ follow from T.

(b) Show that $\forall X_1 \forall X_2 \big(\psi_p(X_1, X_2) \longrightarrow p(X_1, X_2)\big)$ is a tautology.

(c) Derive $\forall X_1 \forall X_2 \big(p(X_1, X_2) \longrightarrow \psi_p\big)$ and show that an inconsistency has occurred.